Examlet A Review Session — CS 173 Staff

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Problem 1

Simplify these expressions:

(a)

$$\frac{\log_3(81^k)}{7k}$$

(b)

$$\log_2(13) \cdot \log_{13}(2048)$$

Problem 2

For the following statement, please state the contrapositive **and** the negation. Move all instances of "not" to individual predicates.

$$\forall x \in \mathbb{R}, f(x) > g(x) \implies f(x+1) < g(x+1)$$

Problem 3

Use direct proof to prove the following claim.

For all $x, y \in \mathbb{R}$ where $x \neq 0$, if x and $\frac{y+1}{3}$ are rational, then $\frac{1}{x} + y$ is rational.

Problem 4

Prove the following claim by contrapositive.

For all $a, b, c \in \mathbb{Z}$ with a and c both non-zero, if $ac \mid bc$ then $a \mid b$.

Problem 5

Recall that $[n]_k$ is the set of all integers congruent to n modulo k. These are the number x such that $k \mid n - x$. Now, simplify the following expression:

$$\left[34^8 + 1600 * 9^{15}\right]_{16}$$

Problem 6

Let $A = \{1,2,3\}$ and $B = \{\{1,2\},3\}$. Compute or simplify the following expressions.

(a)

$$(A - \emptyset) \cap B \cap \mathbb{Z}$$

(b)

$$\emptyset \times B$$

(c)

$$|\emptyset \cup A \cup B|$$

Problem 7

Let $A = \left\{ (a,b) \in \mathbb{R}^2 \;\middle|\; a = 3 - b^2 \right\}$ and $B = \left\{ (x,y) \in \mathbb{R}^2 \;\middle|\; |x| \geq 1 \text{ or } |y| \geq 1 \right\}$. Prove that $A \subseteq B$.

Hint: consider proof by cases on the value of b.

Problem 8

Prove that the following identity is true or give a concrete counterexample.

$$\text{For any sets } A, B \text{ and } C, \text{ if } A \times C \subseteq B \times C, \text{ then } A \subseteq B$$