Relations Tutorial Problems

1. Constructing a concrete relation

Construct a relation R on the set $\{1, 2, 3\}$ such that all the following are true:

- $1R2$
- R is symmetric
- R is transitive
- R is not an equivalence relation

(You are constructing just one relation which satisfies all four conditions, not *a separate relation for each condition. You can specify the relation however you want: a diagram with arrows, a table of related pairs, etc.)*

2. Discussion manual problems

Do the following problems from the discussion manual. (Note that when these problems say something like "Define a relation R on A such that ..."; they mean "*We are hereby defining* a relation R on A such that ...". In particular, it is *not* asking *you* to provide a definition.)

- 4.2 parts (a) and (b)
- 4.3 part (a), except you do not need to prove the relation is an equivalence relation.
- 4.3 part (b)

3. Abstract relation proof

Let R and S be symmetric relations on some set A. Define a relation $\sim$ on A such that $x \sim y$ if and only if xRy and $\neg(xSy)$. Prove that $\sim$ is symmetric.

4. Sorting

(This question is purposefully more open-ended than usual. Don't worry about getting the same answer as us, and move on when you don't have more to profitably discuss.) In programming, sorting a list of numbers in ascending order is sometimes called "sorting by $<$". A good sorting API will allow you to sort by user-defined relations - e.g. if you want all the odds ascending followed by all the evens ascending, this would be sorting by R where R is defined by "aRb iff either a is odd and b is even, or they have the same parity[1] and $a < b$". But there are also relations which are not usable for sorting. In general, what properties should a relation R have (or not have) in order for "sorting by R" to make sense (for example, does R need to be reflexive, symmetric, etc?)?

[1] i.e. they're both odd or both even