Trees and Grammars Discussion Problems

1. Thinking About Trees

Suppose you have a binary tree consisting of 10 nodes.

(a) What is the **tallest** possible way to draw this tree?

    (i) What is the height of this tree?

    (i) How many internal nodes does this tree have? How many leaves?

(b) What is the **shortest** possible way to draw this tree?

    (i) What is the height of this tree?

    (i) How many internal nodes does this tree have? How many leaves?

(c) Can you construct a full binary tree with 10 nodes? Explain informally why or why not.

(d) Write a generalized form to describe the number of nodes, n, that can form a full binary tree.

2. Constructing a Grammar

A *palindrome* is a string that is the same forwards as it is backwards. For example, "racecar," "madam," and "level" are all examples of palindromes.

(a) Design a context-free grammar that generates all possible palindromes involving the symbols "a" and "b". *Explicitly state what start and terminal symbols you are using.*

(b) Modify the grammar from part (a) to only generate palindromes where any "a" symbols must be outside any "b" symbols. More specifically, the grammar should generate strings of the form $a^n b^m a^n$. *Try to use as few rules as possible!*

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