CS 173 Lecture 9a: Two-Way Bounding

Lecture 5b: For sets A & B, $A = B \leftrightarrow A \subseteq B \wedge B \subseteq A$.

More generally: Sometimes trying to prove $X = y$ is hard, but $x \le y$ is easy & $y \le x$ is kind of easy.

Sometimes the number we want cannot be calculated, so we give upper & lower bounds

Example: What is $\sqrt{2}$?

$1^2 = 1$. So $1 < \sqrt{2}$.

$2^2 = 4$. So $2 > \sqrt{2}$

0 is a lower bound

1 is a lower bound.

2 is an upper bound.

3 is an upper bound.

$1.2^2 = 1.44$.

$1.3^2 = 1.69$

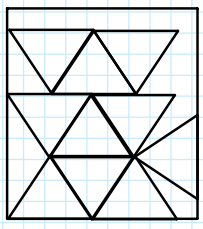
$1.5^2 = 2.25$

$1.3 < \sqrt{2} < 1.5$

1.3: a lower bound; 1.5: an upper bound.

example: 10×10 sheet of paper. Want to cut isoceles triangles of base 4 & height 3.

packing



$m \ge 10$

m = Maximum number of such triangles.

each triangle has area $\frac{1}{2} \cdot 4 \cdot 3 = 6$

sheet of paper has area 100.

$m \le \frac{100}{6} \approx 16.6$

$m \le 16$.

$10 \le m \le 16$.

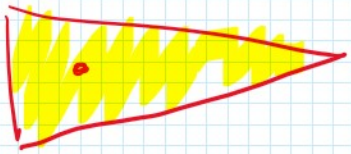
possible.

$10 \leq n \leq 16$.

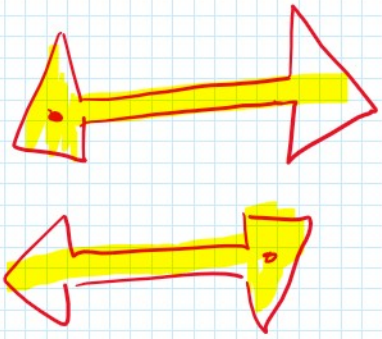
example: Suppose you are told to guard all possible funny-looking rooms w/ n walls using one 360° camera (w/ infinite range)

What is the largest possible n?

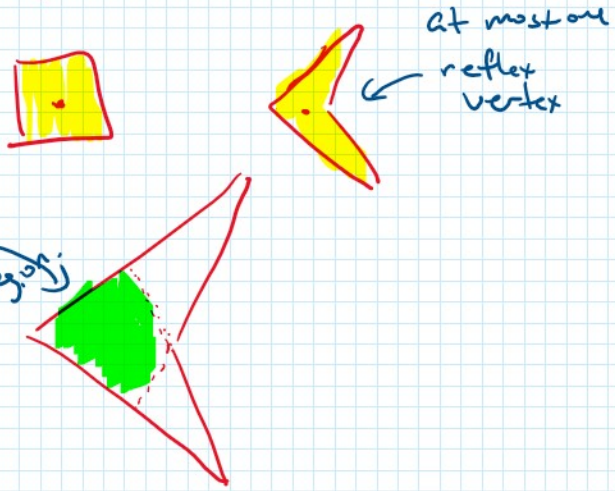
Lower bound: 3



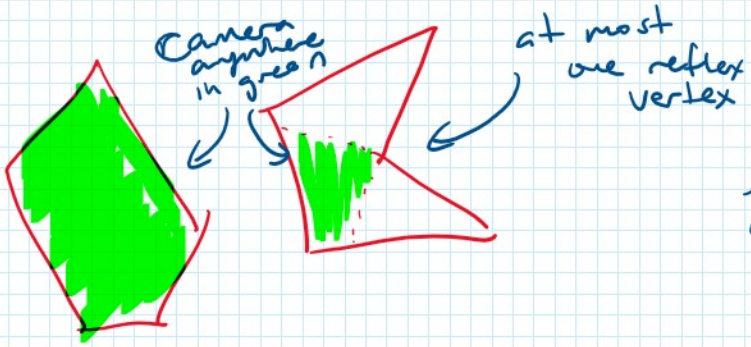
upper bound: ~~10~~ 9



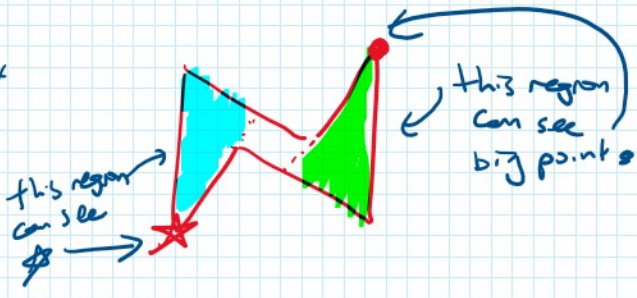
lower bound: 4

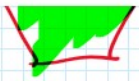


lower bound: 5



upper bound: ~~6~~ 5





$n = 5$

The largest # of walls such that one 360° camera can guard <u>all</u> funny-looking rooms w/ that many walls is 5.