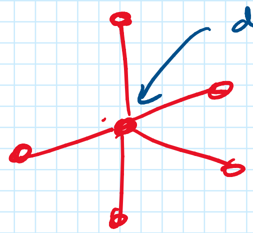
CS 173 Lecture 8b: Properties of Graphs

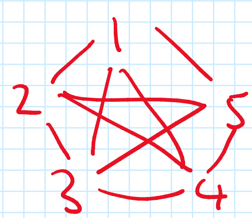
Given a graph $G=(V,E)$, the degree of a vertex $v \in V$, $\deg(v)$ is the number of edges incident to v.



Handshake Lemma

$$\sum_{v \in V} \deg(v) = 2|E|$$

Proof: every edge is incident to exactly two vertices. So counting degree of each vertex counts each edge twice. $\square$



K_5

$V = \{1, .., n\}$

$E = \{\{u,v\} : 1 \le u,v \le n,\ u \ne v\}.$

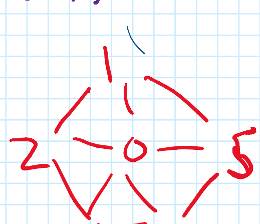
$\forall v \in V,\ \deg(v) = n-1$

So $\sum_{v \in V} \deg(v) = n(n-1) = 2|E|.$

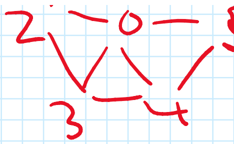
$\Rightarrow |E| = \frac{n(n-1)}{2} = \binom{n}{2}.$

In a graph G,

a walk is a sequence of vertices $v_0, v_1, \dots, v_n$ such that for all i, $0 \le i < n$, $\{v_i, v_{i+1}\} \in E$



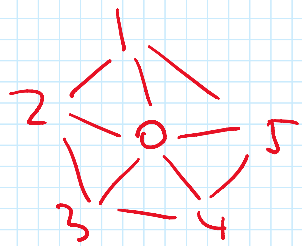
1 2 3 0 1 5 ✓



W_5

1 3 5 0 2 4 ✗

a cycle is a walk that starts & ends at the same vertex, and no other vertices are repeated.



W_5

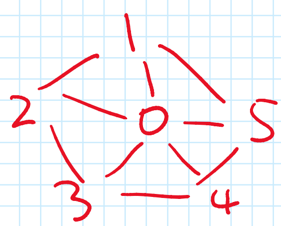
0 1 2 0 ✓

0 1 2 1 0 ✗

1 2 3 4 5 1 ✓

1 3 5 1 ✗

a path is a walk that does not repeat any vertices



W_5

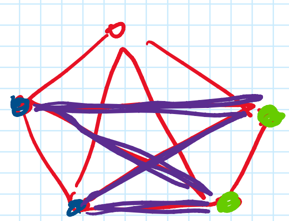
1 2 3 4 5 ✓

3 0 5 4 ✓

2 0 3 2 1 ✗

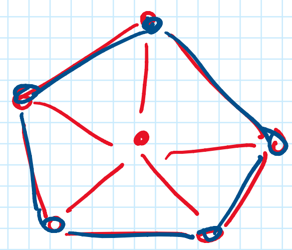
Subgraph.

Given a graph $G = (V, E)$, another graph $H = (V', E')$ is a subgraph of G if $V' \subseteq V$ & $E' \subseteq E$.



K_5

$K_{2,2}$ is a subgraph of K_5.

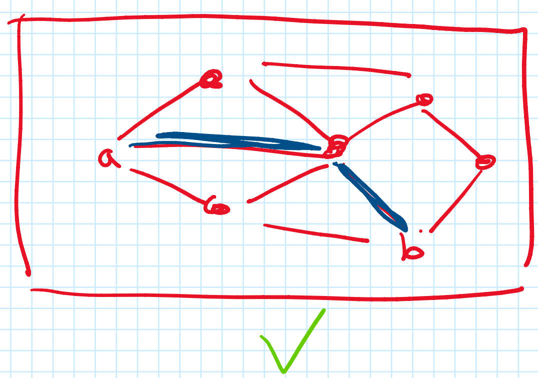


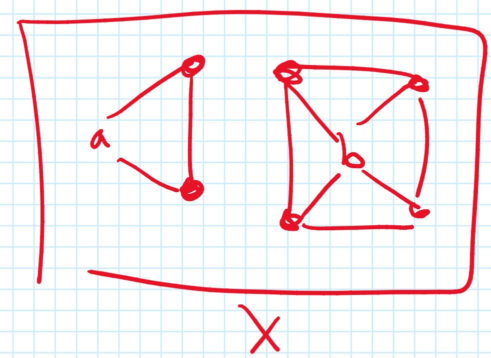
W_5

C_5 is a subgraph of W_5

Connectivity

Given a graph $G=(V,E)$,
vertices a, b are <u>connected</u> if G contains a walk from a to b.

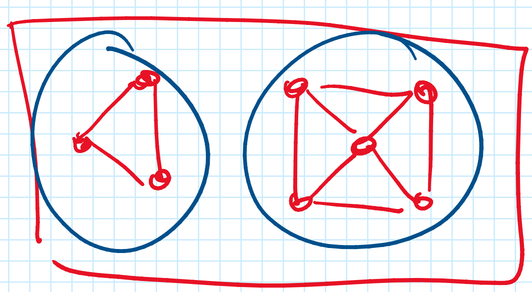




G is connected if for all $a, b \in V$,
a is connected to b.

otherwise, G consists of a bunch of connected components

parts of G that are connected



2 connected components.