CS 173 Lecture 7d: cardinalities & pigeonholes.

Recall that for set A, its cardinality $|A|$ is the number of elements in A.

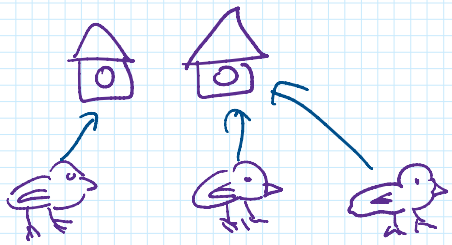
Suppose $f: A \to B$ is onto.
Then $|A| \geq |B|$.

Suppose $f: A \to B$ is one-to-one
Then $|A| \leq |B|$

Suppose $f: A \to B$ is onto & one-to-one.
Then $|A| = |B|$ ($|A| \leq |B|$ & $|B| \leq |A| \to |A| = |B|$ is the CSB Theorem)

Pigeonhole principle.

Suppose $f: A \to B$ such that $|B| < |A|$.
Then f is not one-to-one.
i.e., there exist $x, y \in A$, such that $f(x) = f(y)$.



Claim: Every group of 13 people has two people with the same birth month.

Proof: Let f assign each person their birth month.
Since there are only 12 months,
f assigns two people to the same month. □

Claim: Suppose I have 5 pairs of socks. If I pull out 6 socks, 2 are in a pair.

Proof: Assign each sock to the pair it belongs to
Two socks get assigned to the same pair
since $5 < 6$. □