CS 173 Lecture 7b: Onto functions

or range

Define the image of a function $f : A \to B$ as

$$f(A) = \{ f(a) : a \in A \} \subseteq B.$$

domain

A function $f : A \to B$ is <u>onto</u> if

$$f(A) = B.$$

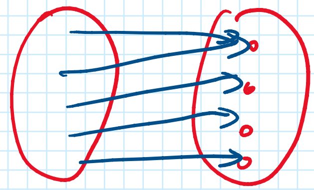
image

codomain

equivalently:

preimage

$$\forall y \in B, \ \exists x \in A, \ f(x) = y.$$



For every student, there is a subject that is the student's primary major.

There is a subject, such that for every student, the subject is the student's major.

Proving that a function is onto:

By definition. Suppose $y \in B$ is arbitrary. Give an example of x s.t. $f(x) = y$.

Claim: $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$, $f(x,y) = 3x - 2y$ is onto.

Scratchwork: Goal: for arbitrary $z \in \mathbb{Z}$, give example of x, y, such that $3x - 2y = z$.

$3x = z + 2y$. What if: $y = z$?

Then $3x = 3z$. So if $x = y = z$,

then $3x - 2y = 3z - 2z = z$.

What if $y = 4z$.

Then $3x = z + 2(4z) = 9z$ so

$x = 3z$.

So $3x - 2y = 3(3z) - 2(4z) = 9z - 8z = z$.

$x = 3z.$

So $3x - 2y = 3(3z) - 2(4z) = 9z - 8z = z.$

Proof. Let $z \in \mathbb{Z}$ be arbitrary. We need to show that there exist $x, y \in \mathbb{Z}$ such that $f(x,y) = z$. Let $x = z$ & $y = z$. Then $f(z,z) = 3z - 2z = z$, as desired. $\square$

Claim: $f: \mathbb{Q} \to \mathbb{Q}$
$f(x) = x/3$ is onto.

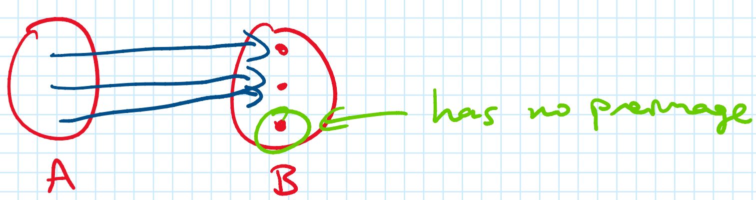
Scratchwork: For $y \in \mathbb{Q}$, find x such that $f(x) = y$.

$x/3 = y, \qquad x = 3y.$

Proof: Let $y \in \mathbb{Q}$ be arbitrary. Let $x = 3y$. Then $f(x) = \frac{3y}{3} = y$. $\square$

Showing f is not onto:

$$\neg(\forall y \in B, \exists x \in A \text{ s.t. } f(x) = y)$$
$$\equiv \exists y \in B\ \neg(\exists x \in A,\ f(x) = y)$$
$$\equiv \exists y \in B, \forall x \in A, \neg(f(x) = y)$$
$$\equiv \exists y \in B, \forall x \in A, f(x) \neq y.$$



Claim: $f: \mathbb{Z} \to \mathbb{Z}$, $f(x) = 2x$, is not onto.

Proof: Suppose $y = 1$. Then if $f(x) = y$, $2x = y$, i.e., $x = y/2$. But $1/2 \notin \mathbb{Z}$. $\square$