CS 173 Lecture 7a: Functions, An Introduction

domain codomain

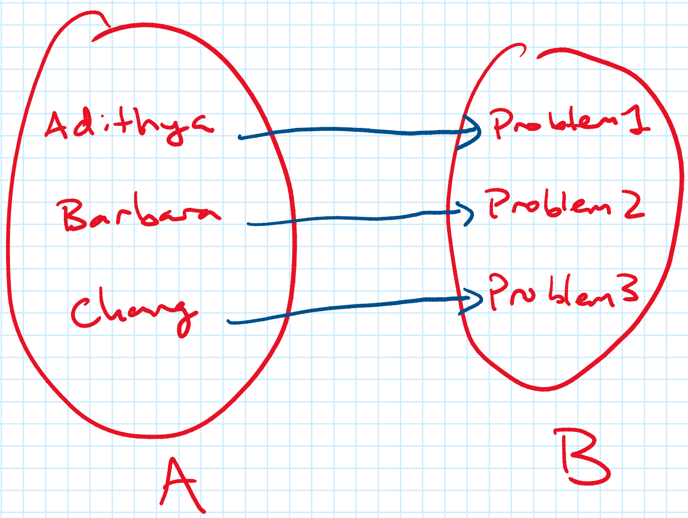
A function $f : A \to B$ assigns to each $x \in A$

type signature input value

exactly one value $f(x) \in B$.

image of x

$f(x) = 2x$ $f(x, y) = x^2 + y^2$



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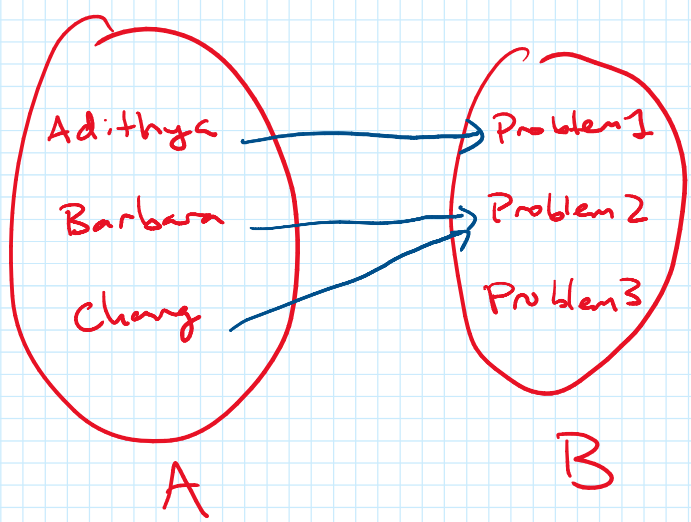

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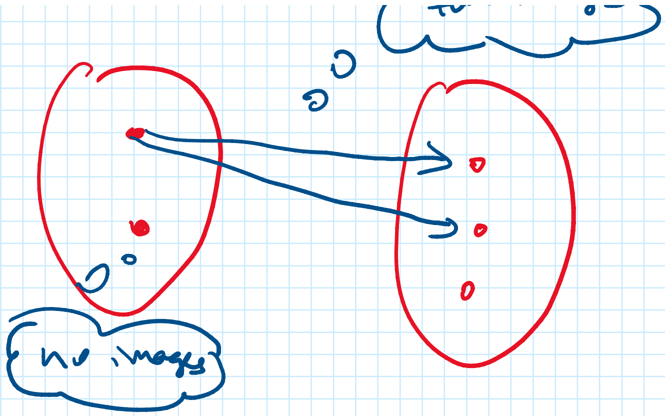
two images



Given a set A, the identity function $id_A : A \to A$

$$id_A(x) = x$$

Given $f: A \to B$ & $g: B \to A$, then f & g are inverses

if $g \circ f = id_A$ & $f \circ g = id_B$.

$g(f(x)) = x$ $\in A$

$f(g(x)) = x$ $\in B$

$f: \mathbb{R} \to \mathbb{R}$

$f(x) = 2x$.

$g: \mathbb{R} \to \mathbb{R}$

$g(x) = x/2$

$$g(f(x)) = x = f(g(x))$$

$$g \circ f = f \circ g = id_{\mathbb{R}}$$

$f: \mathbb{Z} \to \mathbb{Z}$

$f(x) = 2x$

$\exists\, g: \mathbb{Z} \to \mathbb{Z}$ s.t.

$g \circ f = id_{\mathbb{Z}} = f \circ g$.

If g exists then g assigns to $1 \in \mathbb{Z}$

an element $g(1) \in \mathbb{Z}$.

Then $f(g(1)) = 1$.

$f(x)$ is always even.

even integers

even integers

$f(x)$ is always even.

No, such a g cannot exist

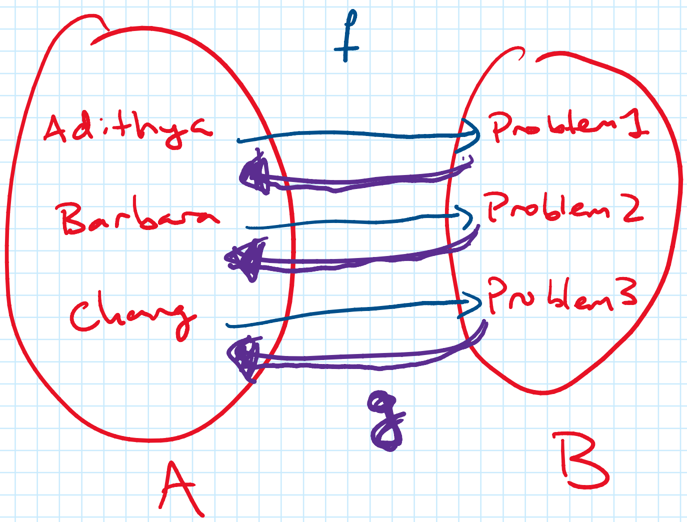
$f: \mathbb{Z} \to 2\mathbb{Z}$

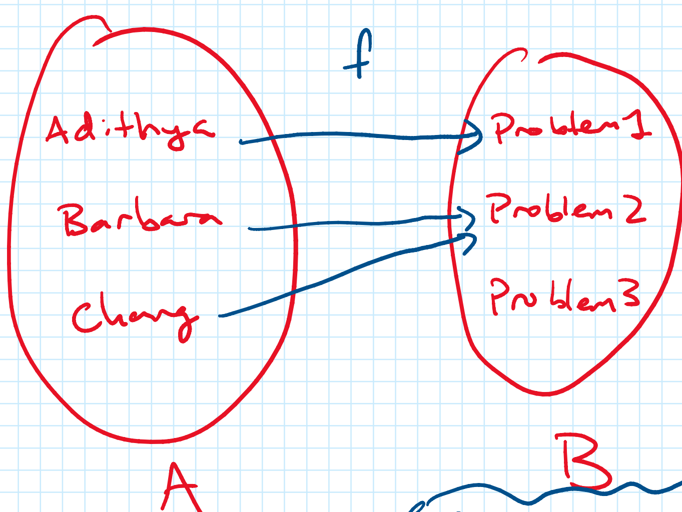
$f(x) = x$

$\exists\ g: 2\mathbb{Z} \to \mathbb{Z}$

s.t. f & g are inverses?

Yes, $g(x) = x/2$.





g(Problem 2) needs to be exactly one student.

Same with g(Problem 3).

Such that $g \circ f = id_A$

& $f \circ g = id_B$.

f is not one-to-one

Problem 1: two students assigned to Problem 2.

Problem 2: no student assigned to Problem 3.

f is not onto

f is not onto

Post-Credits Scene: f has an inverse if and only if f is one-to-one & onto

$\underbrace{\text{ಠ\_ಠ}}_{\text{next time}}$