

# CS 173 Lecture 2a: Existence Proof by Example

There is an integer whose square is not positive

$$\exists x \in \mathbb{Z}, \neg(x^2 > 0). \quad x=0 \rightarrow x^2=0$$

Proof. Let  $x=0$ . Then  $x^2=0$ , which is not greater than 0. This completes the proof  $\square$

Q.E.D.

Quod Erat Demonstratum

negation

$\forall$

For all integers, its square is positive.  $1^2=1 > 0$

Disproof. Let  $x=0$ . Then  $x^2=0$ , which is not positive.  $\leftarrow$  counterexample.  $\square$

$\exists$

Claim: There is an integer  $p$  such that

$$p^2 > 100 \quad \text{and} \quad p < 1.$$

Proof. Let  $p=-100$ . Then  $p^2=10000 > 100$ , and  $p < 1$ .  $\square$

- Be concise / keep it simple.
- There is no "best" example.
- Make the example concrete.

	Existential	Universal
Prove	Example	General Argument
Disprove	General Argument	Counter example