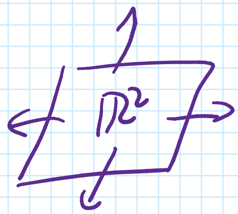
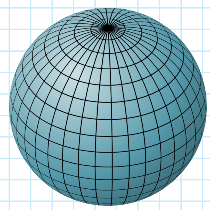
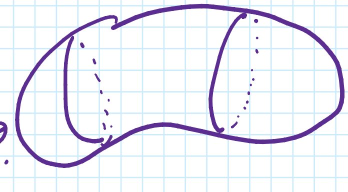
CS 173 Lecture 23b: Surface-Embedded Graphs

Surface: 2d object that "looks" like the plane when zoomed in.



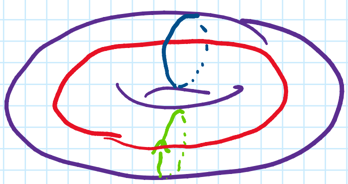


(Wikipedia)

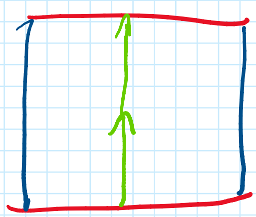


"spheres"

surface of the earth zoomed in looks like the plane.
#: 11:2013

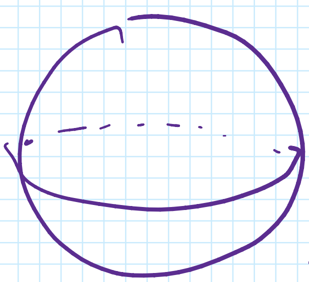


torus

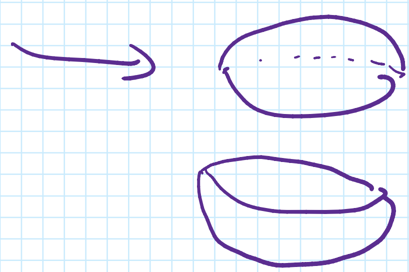


"closed surface"
doesn't go off to ∞

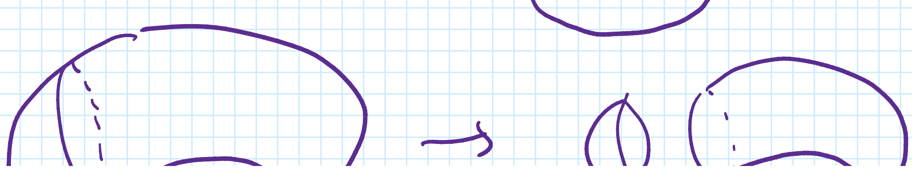
closed surfaces have "genus":
maximum # of times can you cut along in any way the surface w/o disconnecting it

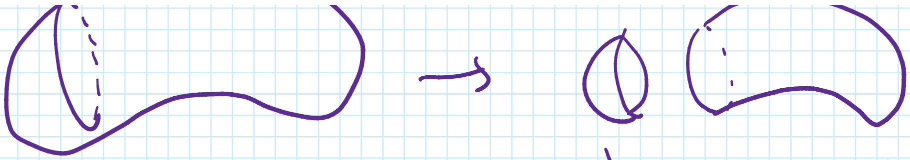


cutting sphere along equator disconnects



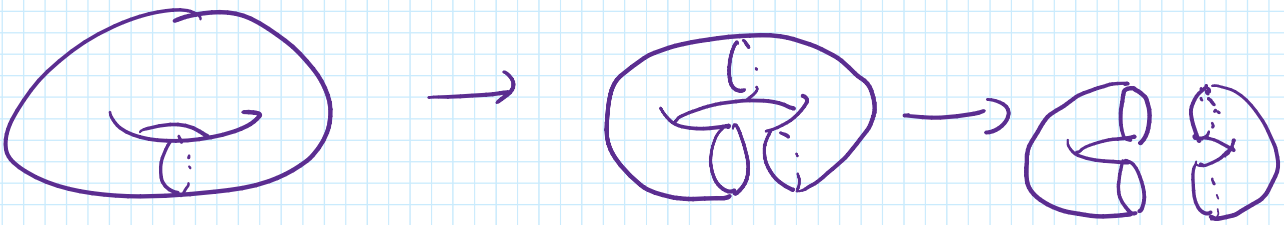
Sphere has genus 0.



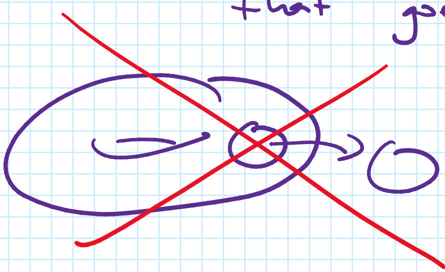


also genus 0.

Topologically, all "genus 0" surfaces are "spheres"



can never disconnect torus w/ one cut that goes around



torus has genus 1.

Thm (Euler's formula) A connected graph drawn on a surface of genus g has $V - E + F = 2 - 2g$.

Proof. By induction. $\blacksquare$

$2E \geq 3F$ is still true.

Cor: $E \leq 3V - 6 + 6g$.

Cor: avg deg $\leq 6 - \frac{12 - 12g}{V}$

K_5 is not planar: $V = 5$ $E = 10 > 9 = 3V - 6$.

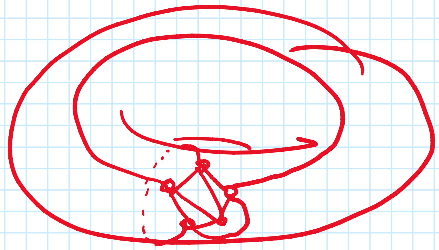
what about the torus? $3V - 6 + 6g = 3V = 15$.

$g = 1$

if G is "toroidal" $E \leq 15$

$g=1$

if G is "toroidal" $E \leq 15$



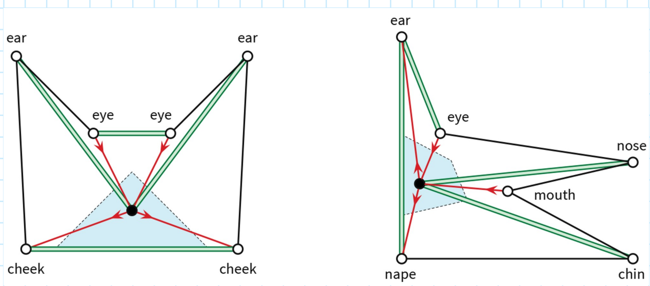
K_5 can be drawn on the torus.

on the torus, can have vertices of deg exactly 6.

in fact K_7 can be drawn on the torus.

$\exists$ of vtx of deg ≤ 5 was fundamental in planar graph algs

$\rightarrow$ difficulty in applying planar graph algs to graphs drawn on $g \geq 1$ surfaces is hard!



from a paper where we tried to show in all instances of a certain problem for toroidal graphs $\exists$ situation "just as good" as having ≤ 5 deg vertex.

A lot of work to see how to deal w/ possibly no vertices of degree ≤ 5!

$\rightarrow$ open, ongoing research.