CS 173 Lecture 20a: Contradiction

p	$\neg p \to F$
T	T
F	F

$\rightsquigarrow$

Proof by contrapositive:
proving p is equivalent to
proving $\neg p \to F$.

Claim: P

Pf: Towards contradiction, suppose not P.

$\vdots$

therefore Q which is false. Therefore P. $\square$

what is false? $x < 3 \wedge x > 17$.
$x \in \mathbb{R} \wedge x^2 < 0$.
Patrick is the Queen of England.

Claim: $\underbrace{\forall a, b \in \mathbb{Z}, \quad a^2 - 4b \neq 2.}_{P}$

Pf: Towards contradiction, suppose
$\underbrace{\exists\, a, b \in \mathbb{Z}, \text{ s.t. } a^2 - 4b = 2.}_{\neg P.}$

$a^2 - 4b = 2 \to a^2 = 2 + 4b$
$\to a^2 = 2(1 + 2b)$
$\to a^2$ is even
$\to a$ is even $\therefore$
$\to \exists k \in \mathbb{Z}, \text{ s.t. } a = 2k.$

contrapositive:
a odd $\to a^2$ odd
Pf. Sketch:
a odd $\to a = 2k+1$
$\to a^2 = 4k^2 + 4k + 1$
$\to a^2 = 2(2k^2 + 2k) + 1$
$\to a^2$ odd

Then $a^2 - 4b = 4k^2 - 4b = 4(k^2 - b) = 2$

$\to 2(k^2 - b) = 1$

Then $a^2 - 4b = 4k^2 - 4b = 4(k^2 - b) = 2$

$\rightarrow 2(k^2 - b) = 1$

$\rightarrow 1$ is even, which is false.

Therefore $\forall a, b \in \mathbb{Z}$, $a^2 - 4b \neq 2$. $\square$

Claim: $\sqrt{5} + \sqrt{13} > \sqrt{34}$

one can plug into a calculator

Pf. Towards contradiction, suppose that $\sqrt{5} + \sqrt{13} \leq \sqrt{34}$.

$\sqrt{5} + \sqrt{13} \leq \sqrt{34} \rightarrow 5 + 2\sqrt{65} + 13 \leq 34$

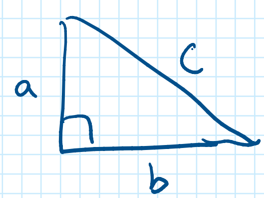
$\rightarrow \sqrt{65} \leq 8$

$\rightarrow 65 \leq 64$

which is false.

Therefore $\sqrt{5} + \sqrt{13} > \sqrt{34}$. $\square$

Claim: There are no right triangles where all three sides have odd-integer lengths.



Pf. Towards contradiction, suppose that T is a triangle w/ side lengths a, b, c where a, b, c are all odd, where a, b are incident to the right angle.

a, b, c are all odd, where a, b are incident to the right angle.

By definition, $a = 2j+1$, $b = 2k+1$, $c = 2\ell+1$ for some $j, k, \ell \in \mathbb{Z}$.

By Pythagorean theorem

$$a^2+b^2=c^2 \rightarrow (2j+1)^2+(2k+1)^2=(2\ell+1)^2$$

$$\rightarrow 4j^2+4j+1+4k^2+4k+1 = 4\ell^2+4\ell+1$$

$$\rightarrow 2\left(2j^2+2j+2k^2+2k+1\right) = 2(2\ell^2+2\ell)+1.$$

i.e. a^2+b^2 is even, c^2 is odd, and $a^2+b^2=c^2$.

This is false. $\square$