CS 173 Lecture 16: Equivalent Propositions & Examples

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\neg p \vee q \equiv p \to q$

DeMorgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q.$$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \to q)$? Tempting: What about $\neg p \to \neg q$? $q \to p$

TRUTH TABLES

p	q	$p \to q$	$\neg(p \to q)$	$\neg p \to \neg q$	$q \to p$
T	T	T	F	T	T
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	F	T	T

$\underbrace{q \to p \equiv \neg p \to \neg q}$

"converse" of $p \to q$ is $q \to p$.

$p \to q \equiv \underbrace{\neg q \to \neg p}$

"contrapositive" of $p \to q$ is $\neg q \to \neg p$.

☆ $\neg(p \to q) \equiv \neg(\neg p \vee q)$

ALGEBRA

$$\neg(p \to q) \equiv \neg(\neg p \vee q)$$
$$\equiv \neg\neg p \wedge \neg q$$
$$\equiv \underline{p \wedge \neg q}$$

contrapositive of $p \to q$ is $\neg q \to \neg p$.

$\neg\neg p \equiv p$

"Think Hard"
"Feynman Method"

Distributive Property:

$$a \wedge (b \vee c) \equiv (a \wedge b) \vee (a \wedge c)$$
$$a \vee (b \wedge c) \equiv (a \vee b) \wedge (a \vee c)$$

Negations w/ quantifiers

for every integer, its square is positive

$\forall x \in \mathbb{Z},\ \underline{x^2 > 0}.$ FALSE $x=0 \to x^2=0.$

$\forall x \in \mathbb{Z},\ \underline{\neg(x^2 > 0)}.$ FALSE $x=1 \to x^2=1>0$

$\exists x \in \mathbb{Z},\ \neg(x^2 > 0)$ TRUE $x=0 \to x^2=0$

$$\neg(\forall x\ p(x)) \equiv \exists x\ \neg p(x)$$
$$\neg(\exists x\ p(x)) \equiv \forall x\ \neg p(x).$$

Examples.

There exists a cookie that has chocolate chips or raisins.

$$\exists\, c \in \text{Cookies},\ \underbrace{\text{choc}(c)}_{c \text{ has chocolate chips}} \vee \underbrace{\text{raisin}(c)}_{c \text{ has raisins}}$$

negation: $\forall c \in \text{Cookies},\ \neg(\text{choc}(c) \vee \text{raisin}(c))$

$\equiv \forall c \in \text{Cookies},\ \neg\text{choc}(c) \wedge \neg\text{raisin}(c)$

Every cookie contains no chocolate chips and no raisins.

For all real numbers x & y, if x & y are both irrational ...

For all real numbers x & y, if x & y are both irrational, then x + y is irrational.

$$\forall x, y \in \mathbb{R}, \ (x \in \mathbb{R}\setminus\mathbb{Q} \wedge y \in \mathbb{R}\setminus\mathbb{Q}) \to (x+y \in \mathbb{R}\setminus\mathbb{Q})$$

$$\exists x, y \in \mathbb{R}, \ x \in \mathbb{R}\setminus\mathbb{Q} \wedge y \in \mathbb{R}\setminus\mathbb{Q} \wedge x+y \notin \mathbb{R}\setminus\mathbb{Q}$$

There exist real numbers x & y such that x & y are irrational but x+y is rational.

and ($\wedge$)

$\pi, \ 4-\pi$

$\pi + (4-\pi) = 4$

p only if q.

if q then p $\equiv$ $q \to p$? No!

$p \to q$