# CS 173 Lecture 19a: Power Set

Recall sets can contain other sets

$\emptyset \subseteq S$ $\forall$ sets $S$, but $\exists$ sets $S$ where $\emptyset \notin S$

e.g. $S = \{$ violin, cello, flute $\}$

$\{\{$ violin, cello $\}$, flute $\} \neq \{$ violin, cello, flute $\}$

Sets containing other sets is useful.

donut shop w/ toppings

toppings $= \{$ sprinkles, chocolate chips, chocolate glaze, strawberry glaze $\}$

valid donuts can be described by subsets:

$$\left\{ \begin{array}{l} \{\text{sprinkles, chocolate glaze}\}, \\ \{\text{chocolate chips, strawberry glaze}\}, \\ \{\text{chocolate glaze, strawberry glaze}\} \end{array} \right\}$$

Power Set of a set $S$

$$\mathcal{P}(S) = \{ T : T \subseteq S \}$$

$$\mathcal{P}(\{0,1\}) = \{ \emptyset, \{0\}, \{1\}, \{0,1\} \}$$

Notice that $|\mathcal{P}(S)| = 2^{|S|}$

for every element in $S$, choose whether or not to put it into a subset.

Power Set can be a domain of a function

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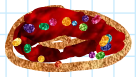
$C : P(\text{toppings}) \to \mathbb{N}$

$C(d)$ = # calories in a donut w/ toppings $d$.

$C(\emptyset) = 500$

$C(\{\text{sprinkles}\}) = 525$

$C(\{\text{sprinkles, chocolate glaze}\}) = 825$



Power Set can also be a codomain

$U$ = set of customers

$f : U \to P(P(\text{toppings}))$

$f(x)$ = set of donut types $x$ likes.

$f(\text{Peter}) = \{\underbrace{\{\text{sprinkles}\}}_{\in P(\text{toppings})}, \underbrace{\{\text{sprinkles, chocolate glaze}\}}_{\in P(\text{toppings})}\}$

$f(\text{Peter}) \subseteq P(\text{toppings})$

i.e. $f(\text{Peter}) \in P(P(\text{toppings}))$ ← set of donuts Amy likes.

$f(\text{Amy}) = \{\{\text{strawberry glaze}\}\} \neq \{\text{strawberry glaze}\}$ ← donut

$f(\text{Amy}) \in P(\text{toppings}) \quad \neq \text{strawberry glaze}$ ← topping