

CS 173 Lecture 14b: More recursion trees

Tree?



	$S(0) = 0$		
	$S(n) = S(n-1) + 1.$		
root: $S(n)$	$l=0$	1	total extra work? 1
internal nodes: recursive calls	$l=1$	1	1
	$l=2$	1	1
		1	
		1	
leaves: base case		0	

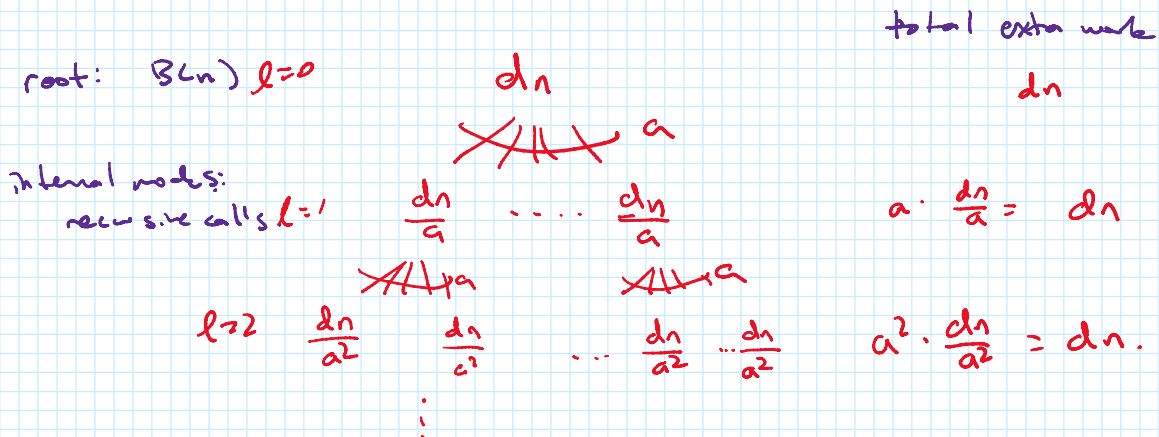
height? $n-h=0. \quad h=n.$

$$\begin{aligned}
 S(n) &= \sum_{l=0}^{h-1} \text{extra work at level } l + \text{leaf-work} \\
 &= \sum_{l=0}^{n-1} 1 + 0 \\
 &= n-1.
 \end{aligned}$$



$$\begin{aligned}
 B(1) &= c \\
 B(n) &= aB(n/a) + dn
 \end{aligned}$$

B defined over powers of $a.$



$$a^2 \quad \frac{1}{a^2} \quad \dots \quad \frac{1}{a^2} \quad \frac{1}{a^2} \quad \dots \quad a \quad \frac{1}{a^2} \quad \dots$$

leaves
base cases $c \dots c \dots c \dots c$ # leaves: a^h

$$\frac{n}{a^h} = 1 \iff h = \log_a n.$$

$$B(n) = \sum_{l=0}^{\log_a n - 1} dn + c a^{\log_a n}$$

$$= dn(\log_a n - 1) + cn.$$



$$C(1) = c$$

$$C(n) = a C\left(\frac{n}{a}\right) + dn^2$$

$l=0:$

$$\frac{dn^2}{a^0} \dots \frac{dn^2}{a^0}$$

$l=1: \left(\frac{n}{a}\right)$

$$\frac{dn^2}{a^2} \dots \frac{dn^2}{a^2}$$

$l=2: \left(\frac{n}{a^2}\right)$

$$\frac{dn^2}{a^4} \dots \frac{dn^2}{a^4}$$

$l=3: \left(\frac{n}{a^3}\right)$

$$\frac{dn^2}{a^8} \dots$$

total extra work

$$dn^2$$

$$a \cdot \frac{dn^2}{a^2} = \frac{dn^2}{a}$$

$$a^2 \cdot \frac{dn^2}{a^4} = \frac{dn^2}{a^2}$$

$$a^4 \cdot \frac{dn^2}{a^8} = \frac{dn^2}{a^4}$$

leaves: a^h

height: $h = \log_a n.$

$$C(n) = dn^2 \sum_{l=0}^{\log_a n - 1} \frac{1}{a^l} + cn$$

$$= (HW)$$

(aside) $\leq dn^2 \sum_{l=0}^{\infty} \frac{1}{a^l} + cn$

$(\dots) \dots \dots \rightarrow a \dots$
 $R \rightarrow \dots$

$$\frac{1}{1-a}$$

$$= \frac{dn^2}{1-a} + cn.$$