CS 173 Lecture 14b: More recursion trees

Tree?

$S(0) = 0$

$S(n) = S(n-1) + 1.$

total extra work?

root: $S(n)$ $\quad \ell = 0 \quad 1 \quad\quad 1$

internal nodes: recursive calls $\ell = 1 \quad 1 \quad\quad 1$

$\ell = 2 \quad 1 \quad\quad 1$

1

$\vdots$

leaves: base case $\quad 0$

height? $\quad n - h = 0. \quad h = n.$

$$S(n) = \sum_{\ell=0}^{h-1} \text{extra work at level } \ell + \text{leaf-work}$$

$$= \sum_{\ell=0}^{n-1} 1 + 0$$

$$= n-1.$$

$B(1) = c$

$B(n) = aB(n/a) + dn$

B defined over powers of a.

total extra work

root: $B(n)$ $\ell=0 \quad dn \quad\quad dn$

a

internal nodes:
recursive calls $\ell = 1 \quad \frac{dn}{a} \quad \cdots \quad \frac{dn}{a} \quad\quad a \cdot \frac{dn}{a} = dn$

$a \quad\quad a$

$\ell = 2 \quad \frac{dn}{a^2} \quad \frac{dn}{a^2} \quad \cdots \quad \frac{dn}{a^2} \quad \cdots \quad \frac{dn}{a^2} \quad\quad a^2 \cdot \frac{dn}{a^2} = dn.$

$\vdots$

$a^2 \quad \cdots \quad a^2 \quad a^2 \qquad a \cdot \frac{\;}{a^2} = dn.$

$\vdots$

leaves
base cases $\quad c \cdots c \quad \cdots \quad c \;\cdots c \qquad$ # leaves: a^h

$$\frac{n}{a^h} = 1 \iff h = \log_a n.$$

$$B(n) = \sum_{l=0}^{\log_a n - 1} dn + c\, a^{\log_a n}$$

$$= dn(\log_a n - 1) + cn.$$

$$C(1) = c$$
$$C(n) = a\,C\left(\tfrac{n}{a}\right) + dn^2$$

total extra work

$l=0$: $\quad dn^2 \quad\quad\quad dn^2$

a

$l=1$: $\left(\frac{n}{a}\right)$ $\quad \frac{dn^2}{a^2} \cdots \frac{dn^2}{a^2} \qquad a \cdot \frac{dn^2}{a^2} = \frac{dn^2}{a}$

$a \qquad a$

$l=2$: $\left(\frac{n}{a^2}\right)$ $\quad \frac{dn^2}{a^4} \cdots \frac{dn^2}{a^4} \qquad a^2 \cdot \frac{dn^2}{a^4} = \frac{dn^2}{a^2}$

$l=3$: $\left(\frac{n}{a^4}\right)$ $\frac{dn^2}{a^8} \cdots \qquad a^4 \cdot \frac{dn^2}{a^8} = \frac{dn^2}{a^4}.$

leaves: a^h
height: $h = \log_a n.$

$$C(n) = dn^2 \sum_{l=0}^{\log_a n - 1} \frac{1}{a^l} + cn$$

$$= (HW)$$

(aside) $\quad \leq dn^2 \sum_{l=0}^{\infty} \frac{1}{a^l} + cn$

$\frac{1}{\;}$

$$\ell=0$$

$$\frac{1}{1-a}$$

$$= \frac{dn^2}{1-a} + cn.$$