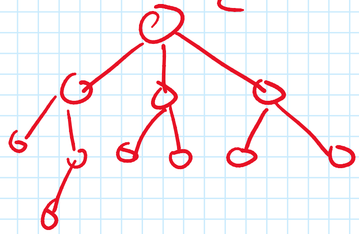
CS 173 Lecture 13a: Trees

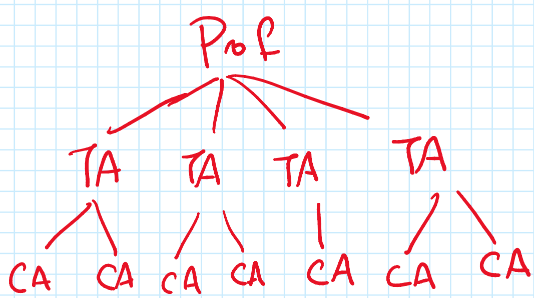
A (rooted) tree an (undirected) acyclic graph w/ a special vertex called the root.

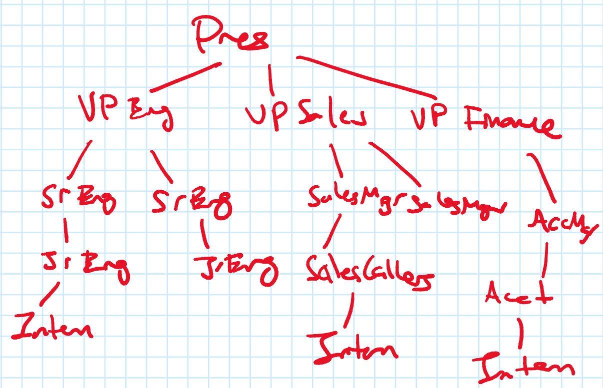
Convention: draw the root at the top



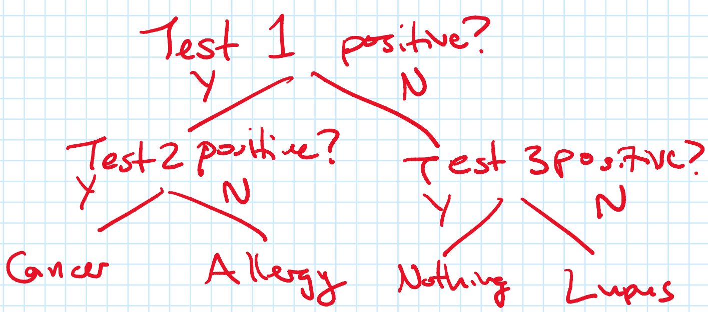
Why trees?

Org Chart

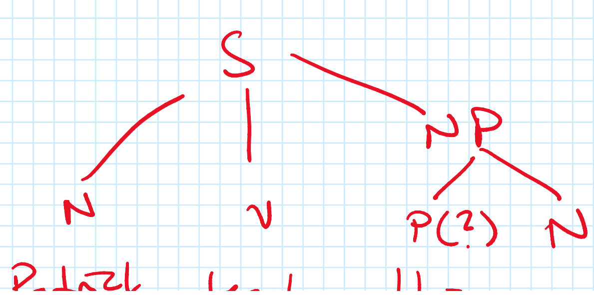




Decision Tree

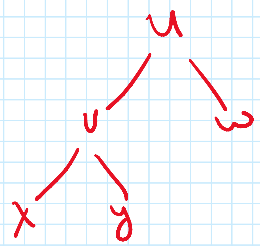


Grammar Tree



N V P(?) N
Patrick teaches this course

Laundry List of words



two incident vertices:
one closer to root: parent
farther away: child

u is v's parent; v is u's child

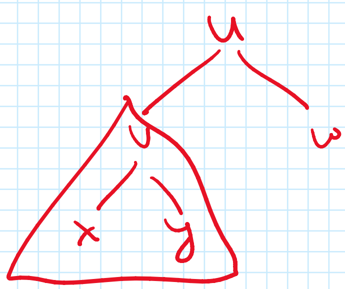
Children of the same parent are siblings
u & w are siblings

vertex w/ no children: leaf
x, y, w are leaves

otherwise: internal.
u, v, w are internal.

given v, vertices on the path root ⇝ v are ancestors of v.
y's ancestors are u & v.

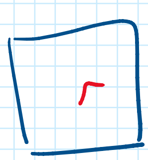
vertices w/ v as an ancestor are descendants
v's descendants are x & y.



↑ subtree rooted at v.

given a vertex a, the subtree rooted at a is the tree consisting of a (as the root) & all descendants (& all edges)

Pop Quiz:

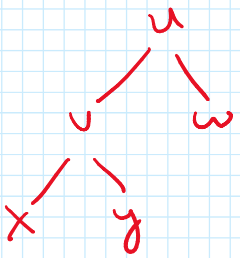


Is r an internal vertex or a leaf?

r (the root) is actually a leaf!

Vertices are divided into levels
level of a vertex v is the length of the unique path root ⇝ v.

level of a vertex v is the length of the unique path root $\rightsquigarrow v$.



— level 0.

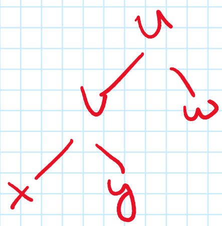
— level 1

— level 2.

The height of a tree T is the max level of any leaf.

height is 2.

Tree is m-ary if every internal node (vertex) has $\leq m$ children,



is a 2-ary tree
binary

an m-ary is full if every internal node has exactly m children, complete if every leaf is at the same level.

⚠! different sources define complete differently

Facts: A full m-ary tree w/ i internal nodes has $mi+1$ nodes

Pf sketch: every node (except the root) is the child of some parent.

Fact: A full & complete binary tree has $2^{h+1}-1$ nodes, where h is the height.

Pf sketch: at level ℓ, 2^ℓ nodes.

$$\sum_{\ell}^{h} 2^\ell = 2^{h+1}-1.$$