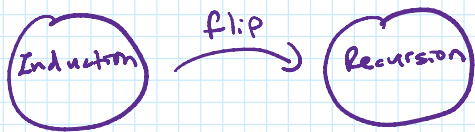


# CS 173 Lecture 12c: Proving Closed Forms



Q<sub>n</sub>:  $E_n = 2E_{n-1} + 2^{n-1}$ ,  $E_0 = 0$ .

Claim:  $E_n = n2^{n-1}$  for  $n \geq 0$

Proof. By Induction.

Base Case:  $E_0 = 0$ .

I.H.: For  $0 \leq k < n$ ,  $E_k = k2^{k-1}$

I.S.:  $E_n = 2E_{n-1} + 2^{n-1}$

$$= 2(n-1)2^{n-2} + 2^{n-1}$$

$$= (n-1)2^{n-1} + 2^{n-1}$$

$$= n2^{n-1}$$

\_\_\_\_\_ X \_\_\_\_\_ □

$T(n) = 2T(\frac{n}{2}) + n$ ,  $T(2) = 1$ ,  $n \geq 2$ ,  $n$  power of 2.

Claim: For  $n \geq 2$  power of 2,  $T(n) = n[(\log_2 n) - \frac{1}{2}]$

Proof. By induction.

Base Case:  $n = 2$ ,  $T(n) = 1$

I.H. for  $2 \leq k < n$ ,  $k$  power of 2,

$$T(k) = k[(\log_2 k) - \frac{1}{2}]$$

I.S.  $T(n) = 2T(\frac{n}{2}) + n$

$$= 2 \cdot \frac{n}{2} [(\log_2 \frac{n}{2}) - \frac{1}{2}] + n$$

$$= n [(\log_2 n - \log_2 2) - \frac{1}{2}] + n$$

$$= n [(\log_2 n) - \frac{1}{2}] - n + n$$

$$= n [(\log_2 n) - \frac{1}{2}]$$

\_\_\_\_\_ X \_\_\_\_\_ □

Odd Fibonacci:  $F(0)=0$ ,  $F(1)=1$   
 $F(n) = F(n-1) + F(n-2)$ .

Claim:  $F(3n+1)$  is odd for all  $n \geq 0$ .

Proof: By Induction.

BC.  $n=0$ :  $F(1)=1$  which is odd.

IH. Assume for  $0 \leq k < n$   $F(3k+1)$  is odd.

IS.  $F(3n+1) = F(3n) + F(3n-1)$ .

$$\begin{aligned} &= F(3n-1) + F(3n-2) + F(3n-1) \\ &= 2F(3n-1) + \overset{\uparrow \text{same}}{F(3(n-1)+1)} \\ &= 2F(3n-1) + \underbrace{(2l+1)}_{\text{odd}} \text{ for some } l \in \mathbb{Z}. \\ &= 2(F(3n-1) + l) + 1. \end{aligned}$$

which is odd. □