CS 173 Lecture 11b: Greedy Coloring, Again

Lemma (Greedy Coloring) Let $D = \max_{v \in V} \deg(v)$.
Then $\chi(G) \le D+1$.

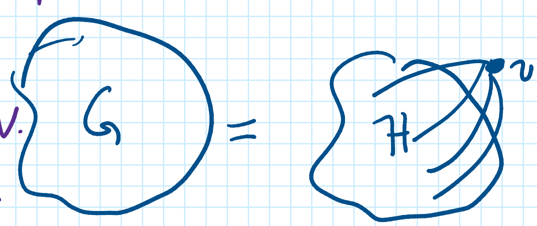
Proof: Let $D \in \mathbb{N}$ be arbitrary. We will prove for all $n \ge 1$

$P(n) =$ "If $G=(V,E)$ is a graph such that $|V|=n$
$\max_{v \in V} \deg(v) \le D$, then $\chi(G) \le D+1$".

Base Case: $n=1$. $E = \emptyset$, and there is only one vertex v. $\max_{v \in V} \deg(v) = 0$,
Also $\chi(G) = 1 \le D+1$.

IH: For $0 \le k < n$, suppose $P(k)$ holds.

I.S.: Suppose $G=(V,E)$ is a graph such that $|V|=n$
& $\max_{v \in V} \deg(v) \le D$.



Pick an arbitrary vertex $v \in V$.
Then let H be the subgraph of G obtained by removing v & its incident edges from G.
H has $n-1$ vertices, and for each vertex u of H,
$\deg_H(u) \le \deg_G(u) \le D$.
So by the I.H., $\chi(H) \le D+1$.
Let $c: V - \{v\} \to \{1, 2, \dots, D+1\}$ be a proper $(D+1)$-coloring of H. Since $\deg(v) \le D$, the set

set of colors used by the neighbors of v

$\circ\circ$ $S = \{ c(u) : uv \in E \}$ has cardinality at most D
We can find a value $k \in \{1, \dots, D+1\} - S$, and let $c' : V \to \{1, \dots, D+1\}$ be defined by

$$c'(u) = \begin{cases} k & \text{if } u = v \end{cases}$$

$$c'(u) = \begin{cases} k & \text{if } u = v \\ c(u) & \text{if } u \neq v. \end{cases}$$

c' is a proper $(D+1)$-coloring of G.

Therefore $\chi(G) \leq D+1$. $\square$