Countability

Part b: To Infinity and Beyond

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Learning Objectives

By the end of this lesson, you will be able to:

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- Define countable and uncountable.

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- Recall standard examples of countable and uncountable sets.

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By the end of this lesson, you will be able to:

- Define countable and uncountable.
- Recall standard examples of countable and uncountable sets.
- Identify whether a given set is countable or uncountable.

Countable Sets

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Definition

An infinite set A is ***countably infinite*** if $|A| = |\mathbb{N}|$.

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Definition

A set is ***countable*** if it is finite or countably infinite.

Fact

If B is countable and $A \subseteq B$, then A is countable.

Uncountable Sets

Uncountable Sets

Definition

A set S is ***uncountable*** if it is not countable.

Uncountable Sets

Definition

A set S is ***uncountable*** if it is not countable.

Do these even exist...?

Cantor's Diagonalization Argument

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Theorem

$\mathbb{P}(\mathbb{N})$ *is uncountable.*

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	b_0	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	$\ldots$
v_0	1	1	0	1	1	0	1	1	1	1	$\ldots$
v_1	1	1	0	0	1	0	1	1	0	0	$\ldots$
v_2	0	0	0	0	1	0	0	1	0	0	$\ldots$
v_3	0	1	1	1	1	0	1	0	0	0	$\ldots$
v_4	0	0	0	0	1	1	1	0	1	1	$\ldots$
v_5	1	1	1	0	1	0	1	0	0	1	$\ldots$
$\ldots$	$\ldots$										

Cantor's Diagonalization Argument

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v_3	0	1	1	1	1	0	1	0	0	0	$\ldots$
v_4	0	0	0	0	1	1	1	0	1	1	$\ldots$
v_5	1	1	1	0	1	0	1	0	0	1	$\ldots$
$\ldots$	$\ldots$										

Theorem

The interval $(0, 1)$ *of real numbers is uncountable.*

More Uncountable Sets

More Uncountable Sets

Fact

If A is uncountable and $A \subseteq B$, then B is uncountable.

More Uncountable Sets

Fact

If A is uncountable and $A \subseteq B$, then B is uncountable.

Theorem

The set of functions from $\mathbb{Z}$ to $\mathbb{Z}$ is uncountable.

Recap: Learning Objectives

By the end of this lesson, you will be able to:

- Define countable and uncountable.
- Recall standard examples of countable and uncountable sets.
- Identify whether a given set is countable or uncountable.