Proof by Contradiction

Part b: More Examples

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Learning Objective

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- Write a proof by contradiction.

Example 1: Anything But Two

Prove $\forall a, b \in \mathbb{Z}$, $a^2 - 4b \neq 2$.

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Prove $\forall a, b \in \mathbb{Z}$, $a^2 - 4b \neq 2$.

Proof.

The proof is by contradiction. Suppose there exist integers a and b such that $a^2 - 4b = 2$.

This is a contradiction. Therefore, $\forall a, b \in \mathbb{Z}$, $a^2 - 4b \neq 2$. □

Example 2: An Irrational Inequality

Prove $\sqrt{5} + \sqrt{13} > \sqrt{34}$.

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Prove $\sqrt{5} + \sqrt{13} > \sqrt{34}$.

Proof.

The proof is by contradiction. Suppose $\sqrt{5} + \sqrt{13} \leq \sqrt{34}$.

This is a contradiction. Therefore, $\sqrt{5} + \sqrt{13} > \sqrt{34}$. □

Example 3: Dense Graphs are Connected

Let $G = (V, E)$ be any graph with $|V| = n$. Prove that if every vertex in G has degree at least $n/2$, then G is connected.

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Let $G = (V, E)$ be any graph with $|V| = n$. Prove that if every vertex in G has degree at least $n/2$, then G is connected.

Proof.

The proof is by contradiction. Suppose every vertex in G has degree at least $n/2$, but G is disconnected.

This is a contradiction. Therefore, G is connected. □

Recap: Learning Objective

By the end of this lesson, you will be able to:

- Write a proof by contradiction.