Collections of Sets

Part b: Partitions

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- Connect the equivalence classes of an equivalence relation on A to parts of a partition of A.

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(3) $S \cap U = \emptyset \; \forall S, U \in \mathcal{P}, S \neq U$ (the sets are pairwise disjoint)

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- Splitting students into k project teams

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- Is $\{[n, n+1) : n \in \mathbb{Z}\}$ a partition of $\mathbb{R}$?

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- Partitioning $\mathbb{Z}$ into congruence classes modulo k
- Define R on $\mathbb{Z}^2$: $(x, y) \; R \; (a, b)$ iff $|x| + |y| = |a| + |b|$.
- Given a partition $\mathcal{P}$ of some set A, define a relation $\sim$ on A by $x \sim y$ iff $\exists S \in \mathcal{P}$ such that $x, y \in S$.

Recap: Learning Objectives

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