

NP

Part b: co-NP and NP-completeness

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Learning Objectives

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- List some examples of NP-complete problems.

Definition

The complexity class **co-NP** (short for *complement is nondeterministic polynomial time*) is the set of all *decision* problems for which you can verify the answer is “no” in polynomial time given a proof/witness/certificate (or, equivalently, the problem’s complement is in NP).

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Example 1: Tautology

Given a Boolean formula with n variables, is it a tautology (i.e., true for all possible variable assignments)?

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Example 1: Tautology

Given a Boolean formula with n variables, is it a tautology (i.e., true for all possible variable assignments)?

Example 2: Complement of Graph Coloring

Given a graph G and an integer k , is it *impossible* to properly color G with k colors?

NP-complete

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- Independent Set
- Traveling Salesman Problem

What we think the world looks like

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