

# Big-O

## Part b: The Formal Definition

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- For specific functions  $f$  and  $g$ , identify whether  $f$  is  $O(g)$  and/or  $\Theta(g)$ .

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$$h(n) = n^2 \quad \text{versus} \quad q(n) = n^3 - 6n^2 + 5n + 20$$

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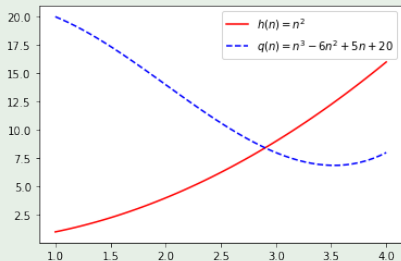
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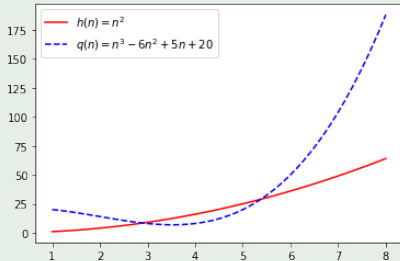
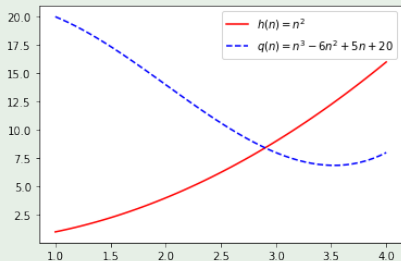
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If  $f(n)$  is  $O(g(n))$  and  $g(n)$  is  $O(f(n))$ , then we say  $f(n)$  is  $\Theta(g(n))$  (and vice versa).

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