More Recursion Trees

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Learning Objective

By the end of this lesson, you will be able to:

- Given a recursively defined function, find its closed form by drawing a recursion tree and adding up the work at all levels.

Example 1: Convenient Cancellation

Define $g : \mathbb{Z}^+ \to \mathbb{Z}^+$ as

$$g(1) = 3$$
$$g(n) = 3g(n/3) + 2n \quad \forall n \geq 3.$$

Find a closed-form for $g(n)$ using a recursion tree. (Assume n is a power of 3.)

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Level	Input	Total Work
0		
1		
2		
k		
h		

Example 2: Changing the Base Case

Let $S = \mathbb{Z}^+ - \{1\}$, and define $q : S \to \mathbb{Z}$ as

$$q(2) = 2$$
$$q(n) = 2q(n-1) + 5 \quad \forall n \geq 3.$$

Find a closed-form for $q(n)$ using a recursion tree.

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Recap: Learning Objective

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