Induction, Episode VI: Return of the I.H.

Part b: Unrolling and Hypercubes

Ian Ludden

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Warning

If you haven't reviewed Section 1.5 of the textbook (Summations), now is a good time.

Unrolling

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By Ludde Lorentz on Unsplash

Unrolling Practice

Example 1: Implicit Summation

Define $g : \mathbb{Z}^+ \to \mathbb{Z}^+$ by

$$g(n) = \begin{cases} 2 & \text{if } n = 1 \\ n(n+1) + g(n-1) & \text{otherwise.} \end{cases}$$

Find a closed-form expression for $g(n)$.

Unrolling Practice

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Example 2: Additive Terms

Define $b : \mathbb{Z}^+ \to \mathbb{Z}^+$ by

$$b(1) = 3$$
$$b(n) = 3b(n-1) + 2n + 1 \ \forall n \geq 2.$$

Find a closed-form expression for $b(n)$.

Hypercubes

Definition

The k-dimensional hypercube, Q_k, is a graph defined recursively for $n \in \mathbb{N}$ by

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- For any $k \geq 1$, Q_k is two copies of Q_{k-1} with edges joining corresponding vertices.

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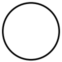

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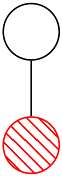

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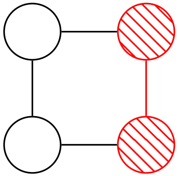

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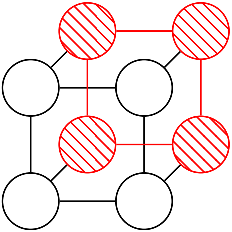

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How many vertices does Q_k have?

Recap: Learning Objectives

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