Proving Properties of Relations

Ian Ludden

Learning Objectives

By the end of this lesson, you will be able to:

Learning Objectives

By the end of this lesson, you will be able to:

- Prove properties about relations (formally).

Learning Objectives

By the end of this lesson, you will be able to:

- Prove properties about relations (formally).
- Prove a relation is (not) a certain type.

Proving Antisymmetry

Proving Antisymmetry

Definition

A relation R on A is ***antisymmetric*** if for all $x, y \in A$ with $x \neq y$, if $x \, R \, y$, then $y \not R \, x$.

Proving Antisymmetry

Definition

A relation R on A is ***antisymmetric*** if for all $x, y \in A$ with $x \neq y$, if $x \, R \, y$, then $y \not R \, x$.

Definition

A relation R on A is ***antisymmetric*** if for all $x, y \in A$, if $x \, R \, y$ and $y \, R \, x$, then $x = y$.

Proving Antisymmetry

Example

Let $A = \mathbb{R}$, and define a relation R on A as $x \, R \, y$ iff $|x| \leq y$. Prove R is antisymmetric.

Proving/Disproving Types of Relations

Example

Let $A = \mathbb{R}$, and define a relation R on A as $x \, R \, y$ iff $x \leq |y|$.
Prove/disprove R is a linear order.

Proving/Disproving Types of Relations

Example

Let $A = \mathbb{Z}^2$, and define a relation $\blacktriangle$ on A as

$$(a, b) \blacktriangle (x, y) \text{ iff } a = x \text{ and } b = y.$$

Classify $\blacktriangle$ as a partial order/linear order/strict partial order/equivalence relation/none of these, and prove your answer.

Recap: Learning Objectives

By the end of this lesson, you will be able to:

- Prove properties about relations (formally).
- Prove a relation is (not) a certain type.