Special Types of Relations

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Learning Objectives

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- Define partial order, linear order, strict partial order, and equivalence relation.

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- Construct the equivalence classes of an equivalence relation.

Recall: Properties of Relations

- Reflexive/irreflexive/neither

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- Reflexive/irreflexive/neither
- Symmetric/antisymmetric/neither

Recall: Properties of Relations

- Reflexive/irreflexive/neither
- Symmetric/antisymmetric/neither
- Transitive/not transitive

Partial Order

Definition

A relation is a ***partial order*** if it is

- reflexive,
- antisymmetric, and
- transitive.

Linear (a.k.a. Total) Order

Definition

A relation is a ***linear order*** if it is

- a partial order, and
- every pair of elements is ***comparable*** ($x \, R \, y$ or $y \, R \, x$).

Strict Partial Order

Definition

A relation is a ***strict partial order*** if it is

- irreflexive,
- antisymmetric, and
- transitive.

Equivalence Relation

Definition

A relation is an ***equivalence relation*** if it is

- reflexive,
- symmetric, and
- transitive.

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Definition

Given an equivalence relation R on a set A, the ***equivalence class*** of x is the set of all elements related to x, denoted

$$[x]_R = \{y \in A : x \, R \, y\} = \{y \in A : y \, R \, x\}.$$

Exercise 1

Classify the relation R on $\mathbb{Z}$ given by $x \, R \, y$ iff $x \equiv y \pmod 7$.

Exercise 2

Classify the relation R on $\mathbb{Z}$ given by $x \; R \; y$ iff $x \mid y$.

Exercise 3

Classify the relation R on $\mathbb{Z}$ given by $x \, R \, y$ iff $x \mid y$ and $x \neq y$.

Exercise 4

Classify the relation R on $\mathbb{Z}$ given by $x \, R \, y$ iff $x \leq y$.

Recap: Learning Objectives

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