Introduction to Relations

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Learning Objectives

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- Represent a relation as a directed graph.
- Restate the formal definitions of standard relation properties and determine which properties a given relation has.

What is a relation?

Definition

A ***relation*** R on a (nonempty) set A is a subset of $A \times A$, that is, a set of ordered pairs of elements from A. We write $x \, R \, y$ (x relates to y) if $(x, y) \in R$ and $x \not R \, y$ (x does not relate to y) otherwise.

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- $A = \mathbb{Z}$, $x\ R\ y$ iff $x \mid y$

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- $A =$ your family, $x \; R \; y$ iff x is a child of y

Properties of Relations: (ir)reflexive

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Neither: Some elements relate to themselves, but some don't.

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Neither: There are some x, y, z satisfying each conditional statement.

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