Set Theory: Laws and Proofs

Ian Ludden

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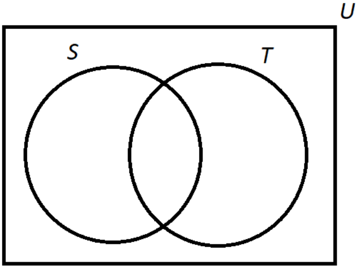

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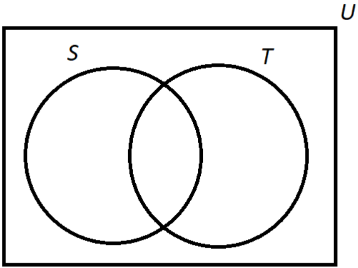
- De Morgan's Laws:



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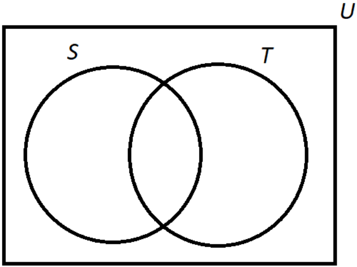

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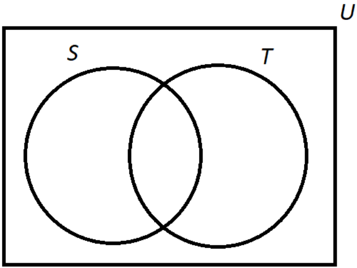

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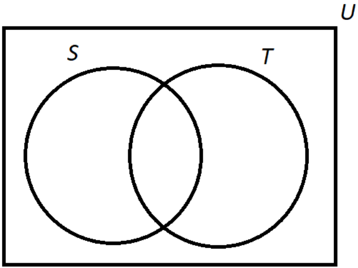
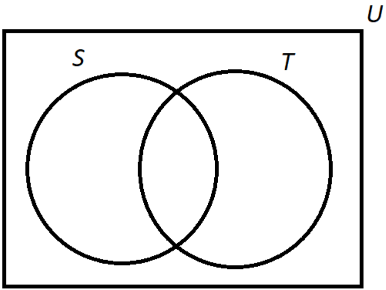
- And many more...



Cardinality after Set Operations



- Size of set union

Cardinality after Set Operations

- Size of set union
- Size of Cartesian product (***product rule***)

Menu

Appetizer	Entree	Dessert
Wings	Pizza	Gelato
Mozz. sticks	Pasta	Rhubarb Pie
Onion rings	Steak	Choc. cake
Salad	Chicken	Cheesecake
Calamari		Cookie
Soup		

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Example

Define $A = \left\{a \in \mathbb{Z} : a^2 - 9 \text{ is odd and } |a| < 25\right\}$ and $B = \{b \in \mathbb{Z} : b \text{ is even}\}$. Prove $A \subseteq B$.

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To prove set equality, show inclusion in both directions

Another Set Proof

Let $A, B, C \subseteq U$. Prove that $(A - B) \subseteq C$ if and only if $(A - C) \subseteq B$.

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Summary of set theory laws:
https://en.wikipedia.org/wiki/Algebra_of_sets