Introduction to Set Theory

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Learning Objectives

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- Compute basic operations on concrete sets.

Definitions via Examples

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  - $\{m \in \mathbb{Z} : 7 \mid (m - 4)\}$

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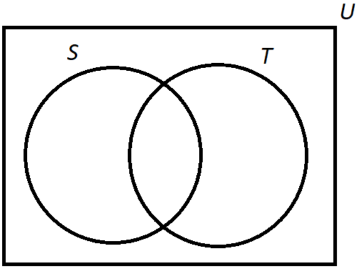
Given sets S and T, we call S a ***subset*** of T (denoted $S \subseteq T$) if every element in S is also an element of T. We call S a ***proper*** subset of T (denoted $S \subset T$) if T has at least one element that S doesn't.

Set Operations

Let S and T be sets in universe U.

- Intersection:
  $S \cap T := \{s : s \in S \land s \in T\}$



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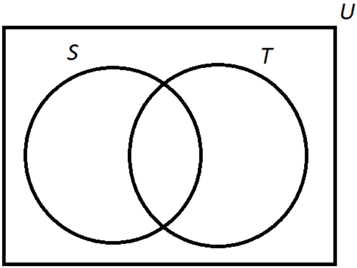

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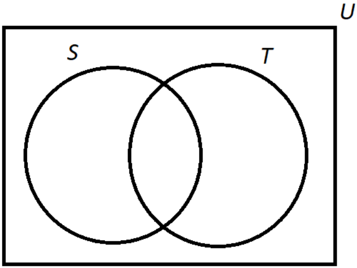

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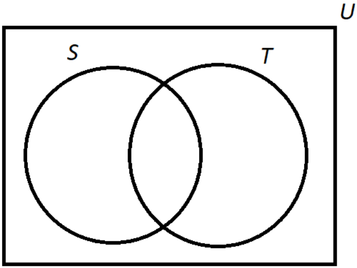

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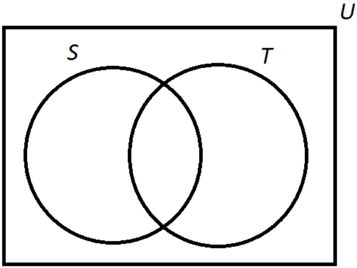
- Cartesian product:
  $S \times T :=$
  $\{(s, t) : s \in S \wedge t \in T\}$



Computing Set Operations

$U := \mathbb{Z}$
$A := \{a \in \mathbb{Z} : a \geq 4 \text{ or } a < 0\}$
$B := \{b \in \mathbb{Z} : b \text{ is odd and } |b| < 6\}$

Recap: Learning Objectives

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