CS 173 Final Review

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Example 1: Tree Induction

A *festive* tree is a full ternary (3-ary) tree with each node colored red or green, such that:

- Every leaf is green.
- If an internal node v has two or three green children, then v is red.
- If an internal node v has zero or one green child(ren), then v is green.

Prove by (strong) induction that every festive tree has strictly more green nodes than red nodes.

Example 2: Abstract Onto Proof

Let $f : \mathbb{N} \to \mathbb{Z}$ be some onto function, and define $g : \mathbb{Z} \to \mathbb{Z}$ by $g(n) = f(|n|) + 4$. Prove that g is onto.

Example 3: Equivalence Relation Proof

Define $\sim$ on $\mathbb{Z}$ so that $x \sim y$ iff $7 \mid (5x + 9y)$. Prove $\sim$ is an equivalence relation.

Example 4: Abstract Subset Proof

Let X, Y, and Z be sets. Prove $(X - Y) \cap (X - Z) \subseteq X - (Y \cap Z)$, and disprove the reverse containment using a concrete counterexample.

Example 5: Proof by Contrapositive (Disc. 1.3(c))

Prove by contrapositive: For all integers m and n, if mn is even, then m is even or n is even.