# Countability Part b: To Infinity and Beyond

Ian Ludden

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• Define countable and uncountable.

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- Define countable and uncountable.
- Recall standard examples of countable and uncountable sets.

- Define countable and uncountable.
- Recall standard examples of countable and uncountable sets.
- Identify whether a given set is countable or uncountable.

## **Countable Sets**

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### **Countable Sets**

#### Definition

An infinite set *A* is *countably infinite* if  $|A| = |\mathbb{N}|$ .

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#### Definition

An infinite set *A* is *countably infinite* if  $|A| = |\mathbb{N}|$ .

#### Definition

A set is *countable* if it is finite or countably infinite.

Al=n, neN N= 2+  $\mathcal{L}(n) = n + 1$  $a_{0}, q_{1}, \dots, q_{n-1}$ a, a, an and 1, ... ) -- an = f(n) YneN < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

#### Definition

An infinite set *A* is *countably infinite* if  $|A| = |\mathbb{N}|$ .

#### Definition

A set is *countable* if it is finite or countably infinite.

#### Fact

*If B is countable and*  $A \subseteq B$ *, then A is countable.* 

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## Uncountable Sets

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### **Uncountable Sets**

#### Definition

A set *S* is *uncountable* if it is not countable.

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### **Uncountable Sets**

#### Definition

A set *S* is *uncountable* if it is not countable.

Do these even exist ...?



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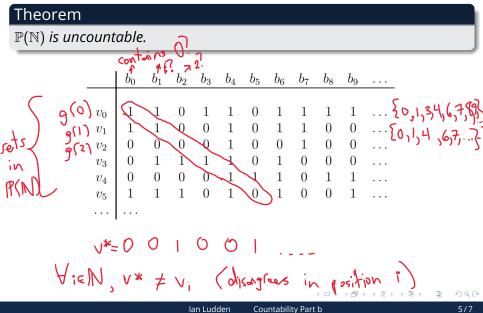
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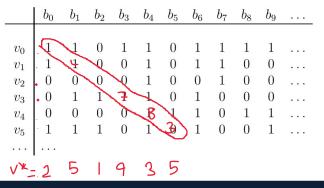
#### Theorem

P(N) is uncountable. set of all subsets of N Proof by contradiction. Suppose P(IN) is countably infinite. Then Ebij- g: N->P(N). g(n) is a subset of N. Well get a contradiction by showing g is not onto. Build set SEN a follows: YKEN, Les iff K&g(k). Then Stylin YnEN. So S has no pre-image  $|N| \neq |\mathbb{P}(N)|$  :  $f(N) \neq \mathbb{P}(N)$ lan Ludden Countability Part b 5/7



#### Theorem

 $\mathbb{P}(\mathbb{N})$  is uncountable.



Theorem

The interval (0,1) of real numbers is uncountable.

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## More Uncountable Sets

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## More Uncountable Sets

#### Fact

If A is uncountable and  $A \subseteq B$ , then B is uncountable.

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## More Uncountable Sets

#### Fact

*If A is uncountable and*  $A \subseteq B$ *, then B is uncountable.* 

#### Theorem

The set of functions from  $\mathbb Z$  to  $\mathbb Z$  is uncountable.

- y the end of this lesson, you will be able to: Define countable and uncountable. Recall standard examples of countable and uncountable sets.
- Identify whether a given set is countable or uncountable.