State Diagrams

Part c: Transition Functions and Counting States

Ian Ludden

Learning Objectives

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Consider a state diagram with a set of states S, a start state $s_0 \in S$, end state(s) $Q \subseteq S$, a set of actions A, and a transition function δ. Formally, δ has type signature $\delta : S \times A \to \mathbb{P}(S)$.

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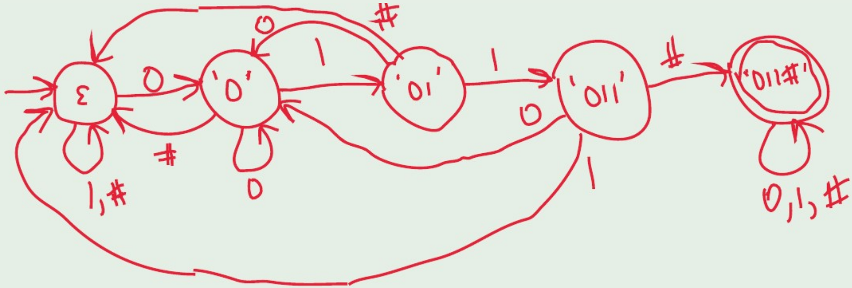
Example: Garage Door Keypad



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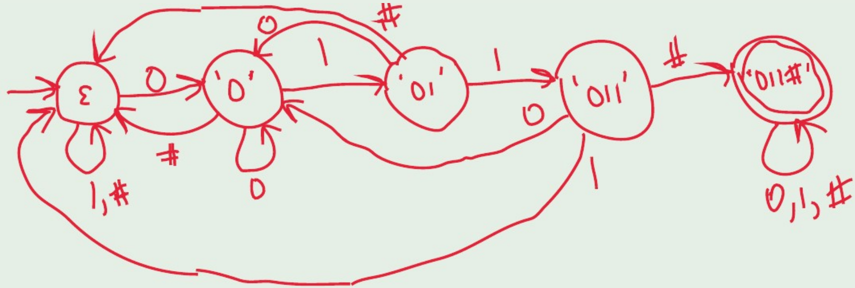

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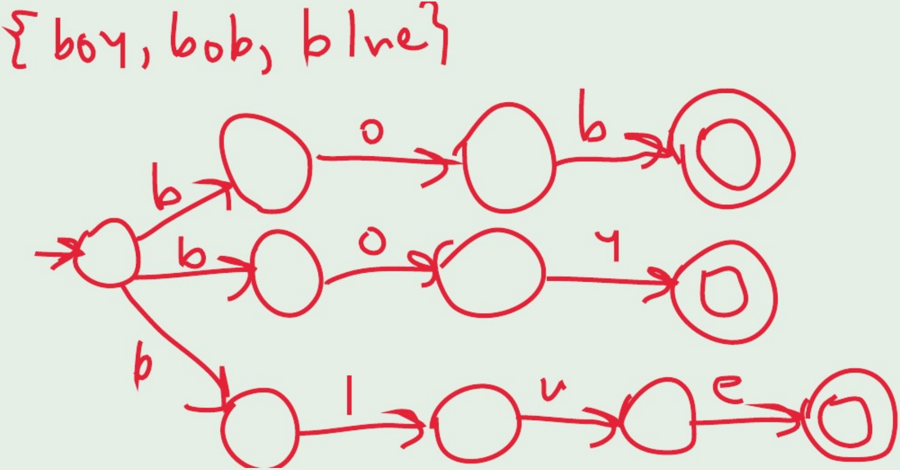
Example 2: Phone Lattice



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- Option 4: Get fancy with hash functions (hash tables, dictionaries, etc.)

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- under normal operation?
- if the digits are not restricted to valid times?

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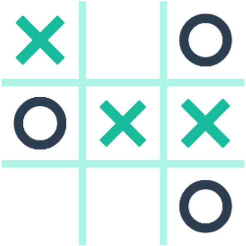

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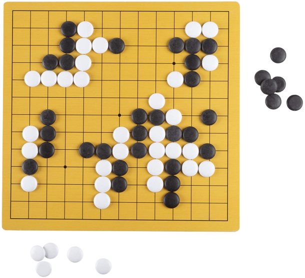

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Fun fact: There are over 43 quintillion (4.3×10^{19}) permutations of the 3×3 Rubik's Cube. (Link to source)

Recap: Learning Objectives

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