

Proof by Contradiction

Part a: The General Method

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Learning Objective

By the end of this lesson, you will be able to:

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- Outline a proof by contradiction.

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You want to prove proposition p , but first attempt at a direct proof isn't working. Backup plan?

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② $T \rightarrow p \equiv \neg p \rightarrow F$ (contrapositive)

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The more obviously false F is, the better. Examples:

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- $0 = 1$
- Purdue's marching band is better than the Marching Illini.

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- 3 Take logical steps that end at a logical contradiction.
- 4 “This is a contradiction. Therefore, [p].”

A Worked Example

Prove $\sqrt{3}$ is irrational.

Proof.

The proof is by contradiction. Suppose $\sqrt{3}$ is rational.

This is a contradiction. Therefore, $\sqrt{3}$ is irrational. □

Recap: Learning Objective

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