# Collections of Sets

## Part b: Partitions

Ian Ludden

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- Connect the equivalence classes of an equivalence relation on $A$ to parts of a partition of $A$.

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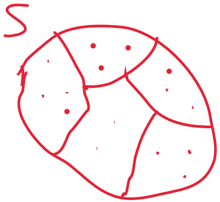

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(3) $S \cap U = \emptyset \ \forall S, U \in \mathcal{P}, S \neq U$ (the sets are pairwise disjoint)

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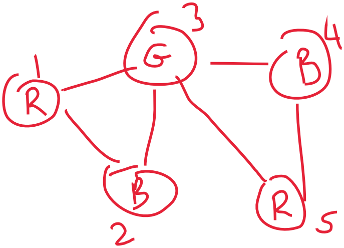
- Color classes of a graph with a proper $k$-coloring

$P = \{\{1,5\}, \{2,4\}, \{3\}\}$

is a partition of $V$

"bipartite"

two



# Examples of Partitions

Partition rules: (1) covers set ✓, (2) non-empty ✓, (3) pairwise disjoint ✓

- Color classes of a graph with a proper *k*-coloring
- Splitting students into *k* project teams

$s_1, s_2, \ldots, s_n$

team 1

$\{\{s_2, s_5, s_8\}, \ldots\}$

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(1) ✓ (2) ✓ (3) ✓ Yes

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- Define $Q_1 = \{(a, b) \in \mathbb{R}^2 : a \geq 0 \text{ and } b \geq 0\}$,
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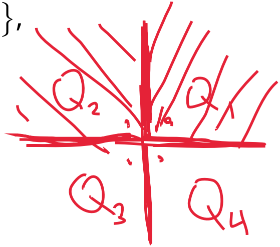
  Is $\{Q_1, Q_2, Q_3, Q_4\}$ a partition of $\mathbb{R}^2$?

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  Is $\{Q_1, Q_2, Q_3, Q_4\}$ a partition of $\mathbb{R}^2$?
- Is $\{[n, n+1) : n \in \mathbb{Z}\}$ a partition of $\mathbb{R}$?

# Partitions and Equivalence Classes

By design, equivalence classes form a partition of their set.

- Partitioning $\mathbb{Z}$ into congruence classes modulo $k$

$k=5$: $\{[0], [1], [2], [3], [4]\}$

(1) ✓ (2) ✓ (3) ✓

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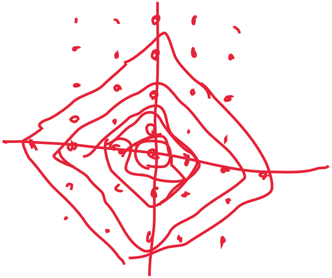

R: $|x| + |y| = |x| + |y|$ ✓

S: ✓

T: ✓

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- Define $R$ on $\mathbb{Z}^2$: $(x, y)\ R\ (a, b)$ iff $|x| + |y| = |a| + |b|$.
- Given a partition $\mathcal{P}$ of some set $A$, define a relation $\sim$ on $A$ by $x \sim y$ iff $\exists S \in \mathcal{P}$ such that $x, y \in S$.

R: ✓

S: ✓

T: ✓

# Recap: Learning Objectives

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