Collections of Sets

Part a: Sets Containing Sets

Ian Ludden

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- Interpret definitions for functions whose input and/or output values are sets (i.e., when the domain and/or co-domain is a power set).

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- Interpret set-builder definitions of sets containing other sets.
- Interpret definitions for functions whose input and/or output values are sets (i.e., when the domain and/or co-domain is a power set).
- Write simple proofs involving collections of sets and/or functions whose input and/or output values are sets.

Sets of Sets

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Example 1

$A = \{☺, ☹, \{0, 2, 4\}, \{1, 3\}, \mathbb{Z}\}$
$B = \{0, \mathbb{Z}^+, ☺, \{2, 4\}, \{1, 3\}\}$

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- $|A \times B| =$

We have to go deeper

“A ~~dream~~ set within a ~~dream~~ set within a ~~dream~~ set”

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(Inception Image Source)

Sets of sets of sets of...

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Example 2

$A = \{\{1,4\},\{5,7,8\},\{3\},\{\emptyset,\{2\}\}\}$
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Definition

Given a set A, the ***power set*** of A, denoted $\mathbb{P}(A)$, is the set of all subsets of A. That is,

$$\mathbb{P}(A) = \{S : S \subseteq A\} .$$

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- $\mathbb{P}(\emptyset) =$

Power Sets as Function (Co-)Domains

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Given a graph $G = (V, E)$, let $n : V \to \mathbb{P}(V)$ such that $n(v)$ is the set of vertices adjacent to v (not counting v, since we assume no self-loops).

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Example 6

Given a graph $G = (V, E)$, let $f : \mathbb{P}(V) \to \mathbb{N}$ such that $f(S) = |\{uv \in E : u, v \in S\}|$.

Proofs with Sets of Sets

Example 7

Let $g : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ such that $g(n) = \{q \in \mathbb{Z}^+ : n \mid q\}$.

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Suppose $g(a) = g(2) \cap g(7)$. What is a? Justify your answer.