NP

Part b: co-NP and NP-completeness

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Learning Objectives

By the end of this lesson, you will be able to:

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- Define co-NP and NP-completeness.

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- List some examples of NP-complete problems.

co-NP

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Definition

The complexity class **co-NP** (short for *complement is nondeterministic polynomial time*) is the set of all *decision* problems for which you can verify the answer is “no” in polynomial time given a proof/witness/certificate (or, equivalently, the problem’s complement is in NP).

$\text{co-NP} \neq \overline{\text{NP}}$ $\quad$ $\text{co-NP} = \{ q : \bar{q} = p \in \text{NP} \}$

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Example 1: Tautology

Given a Boolean formula with n variables, is it a tautology (i.e., true for all possible variable assignments)?

$(x_1 \vee \overline{x_2} \vee x_5) \wedge (\overline{x_3} \vee x_4 \vee x_6)$

If "no": $\exists$ assignment for which False.

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Example 1: Tautology

Given a Boolean formula with n variables, is it a tautology (i.e., true for all possible variable assignments)?

in NP

Example 2: Complement of Graph Coloring

Given a graph G and an integer k, is it *impossible* to properly color G with k colors? witness for "no": a proper k-coloring of G.

NP-complete

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Definition

A decision problem is ***NP-complete*** if

(1) it is in NP, and

(2) if it can be solved in polynomial time, then *every* problem in NP can be solved in polynomial time.

q

NP-complete: think "hard"

q is easy (in P) $\rightarrow$ p is easy (in P) $\forall p \in NP$

solver for q

$\exists p \in NP.\ p \notin P \rightarrow q \notin P.$

$P \neq NP$

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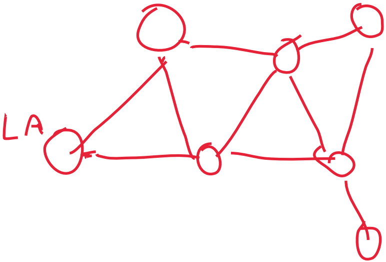
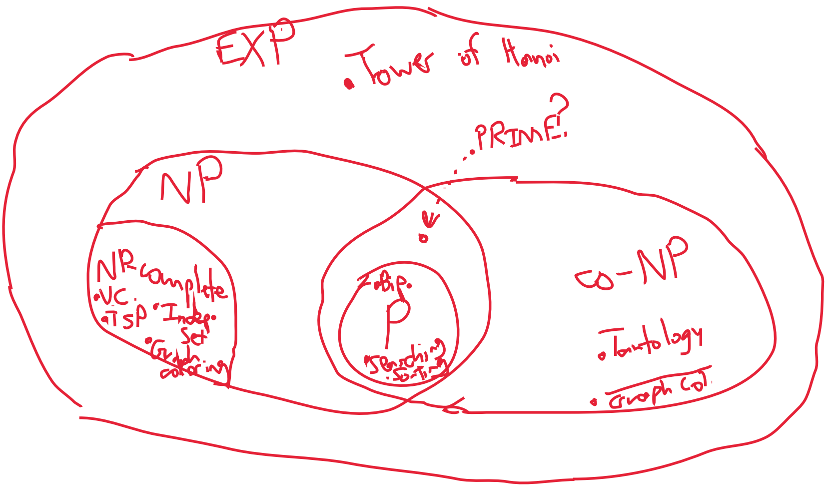
- Circuit SAT
- Vertex Cover
- Independent Set
- Traveling Salesman Problem



What we think the world looks like



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