NP

Part a: EXP, P, and NP

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- Define EXP, P, and NP.
- Know that problems in NP can be solved in exponential time, but it's not known whether they can all be solved in polynomial time.

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Definition

The complexity class **EXP** (short for *exponential time*) is the set of all problems that can be solved in exponential* time in the input size (or faster).

$O(2^{p(n)})$ for some polynomial $p(n)$

$O(4^n) = O((2^2)^n)$

Categorizing Problems

$O(n+m)$

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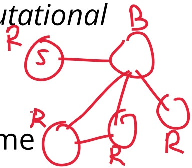

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The complexity class **P** (short for *polynomial time*) is the set of all problems that can be solved in polynomial* time in the input size (or faster).

$n \log n << n^2$ $\quad O(n^{173})$ $\quad$ Bipartite?

Searching/sorting: $O(n), O(n \log n)$

What is NP?

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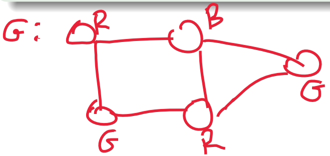
Example 1: Graph Coloring

Given a graph G and integer k, does G have a proper coloring with k colors?



$k=3$

$f: V \to \{R, G, B\}$

$O(m)$ time to verify

NP, continued

NP, continued

Theorem

$P \subseteq NP \subseteq EXP$

$P \subseteq NP$: ignore proof, solve ourselves

$NP \subseteq EXP$: Guess the proof/witness

$f: V \to \{R, G, B\}$

$P = NP$?

$P \neq NP$?

$O(3^n \cdot m)$

NP, continued

Theorem

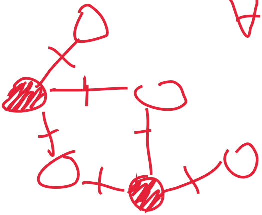
$P \subseteq NP \subseteq EXP$

Example 2: Vertex Cover

Given a graph G and integer k, does G have a vertex cover of size k?

Vertex cover: $S \subseteq V$ is a V.C. iff

$\forall e \in E$, e has an endpoint in S.



Friend: S is a V.C. }

① Check $|S| \leq k$

② Check S is a V.C.

More Problems in NP

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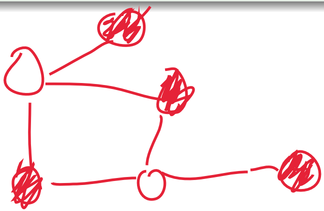
Example 3: Independent Set

Given a graph G and integer k, does G have an independent set of size k?



$S \subseteq V$ is an indep. set if there are no edges w/ both endpoints in S.

More Problems in NP

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Given a graph G and integer k, does G have an independent set of size k?

Example 4: Circuit SAT

Given a Boolean circuit, is there an assignment to the inputs that makes the output true/1?

2^n

Proof: the assignment $O(n)$

Recap: Learning Objectives

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$P \stackrel{?}{=} NP$ Conj: $P \subsetneq NP$

$P = NP$