NP

Part a: EXP, P, and NP

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- Define EXP, P, and NP.
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Example 1: Graph Coloring

Given a graph G and integer k, does G have a proper coloring with k colors?

NP, continued

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Theorem

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Example 2: Vertex Cover

Given a graph G and integer k, does G have a vertex cover of size k?

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Example 4: Circuit SAT

Given a Boolean circuit, is there an assignment to the inputs that makes the output true/1?

Recap: Learning Objectives

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