Big-O

Part b: The Formal Definition

Ian Ludden

Learning Objectives

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Given functions $f, g : \mathbb{N} \to \mathbb{R}$, we say $f(n)$ is $O(g(n))$ if (and only if)

$$\exists \underline{c, k \in} \mathbb{R}^+ \; \forall n \geq k, \; 0 \leq f(n) \leq c \cdot g(n).$$

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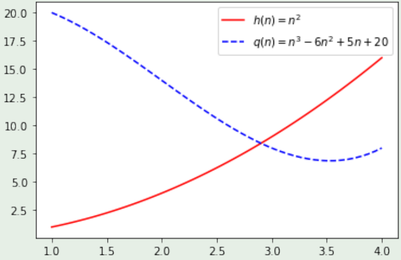

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$\forall c, k \in \mathbb{R}^+, \exists n \geq k, \; f(n) > c \cdot g(n).$

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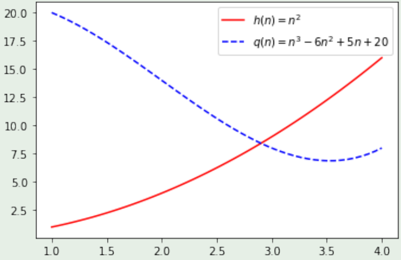
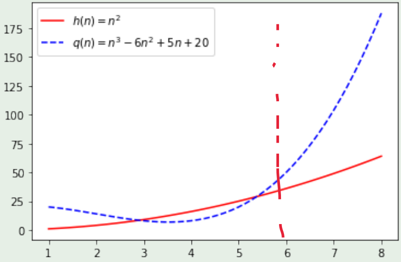
h(n) is O(q(n))





c=1
k=6

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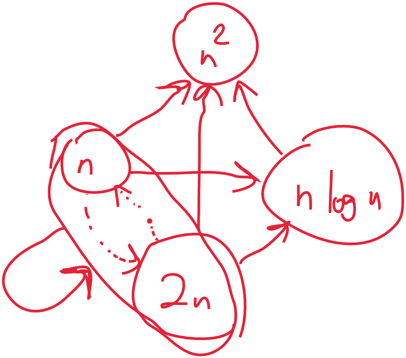
refl., antisymm., transitive

irrefl., anti., transit.



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If $f(n)$ is $O(g(n))$ and $g(n)$ is $O(f(n))$, then we say $f(n)$ is $\Theta(g(n))$ (and vice versa).

$$[n^2] = \{ n^2, \frac{n^2}{5}, 3n^2 + \log n - 7, \ldots \}$$

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