Big-O

Part a: Asymptotic Analysis of Functions

Ian Ludden

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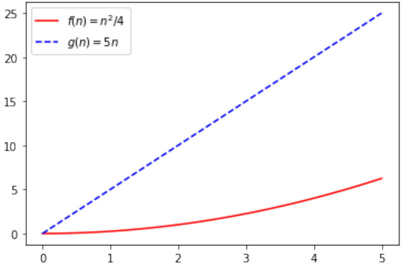

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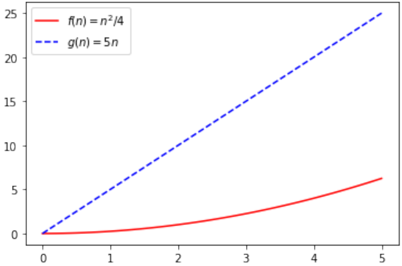

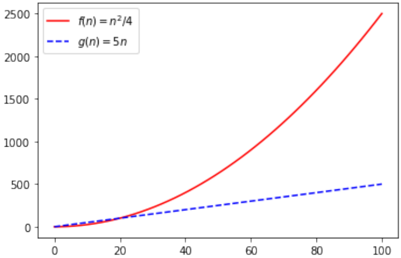

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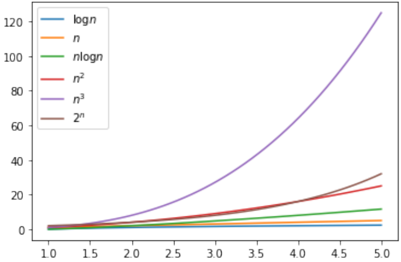

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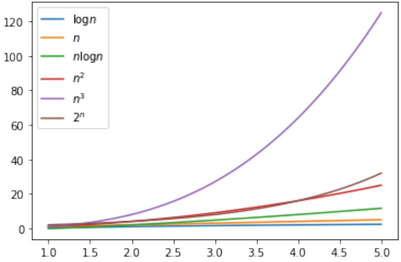

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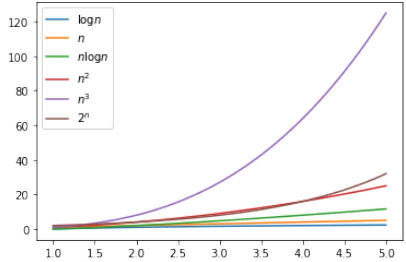

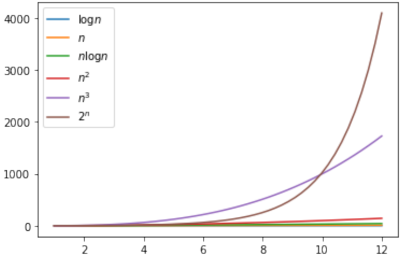

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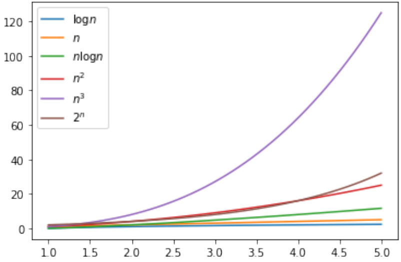

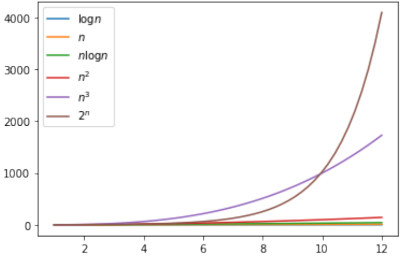

$$1 \ll \log n \ll n \ll n \log n \ll n^2 \ll n^3 \ll 2^n \ll 3^n \ll n!$$

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