Inductive Proofs with Grammar Trees

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Learning Objective

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- Prove properties of context-free grammars by induction with parse trees.

Example 1: One More One

Let G be the grammar defined by start symbol S, terminals 0 and 1, and rules

$$S \to 0S1 \mid 1S0 \mid 1.$$

Prove by (strong) induction that all strings generated by G have more 1's than 0's. Use $\#0(T)$ and $\#1(T)$ as shorthand for the number of 0's and 1's in a tree T.

Example 2: Odd Number of a's

Let G be the grammar defined by start symbol S, terminals $\{a, b\}$, and rules

$$S \to SabS \mid ab.$$

Prove by (strong) induction that all strings generated by G have an odd number of a's.

Recap: Learning Objective

By the end of this lesson, you will be able to:

- Prove properties of context-free grammars by induction with parse trees.