Grammar Trees

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Definition

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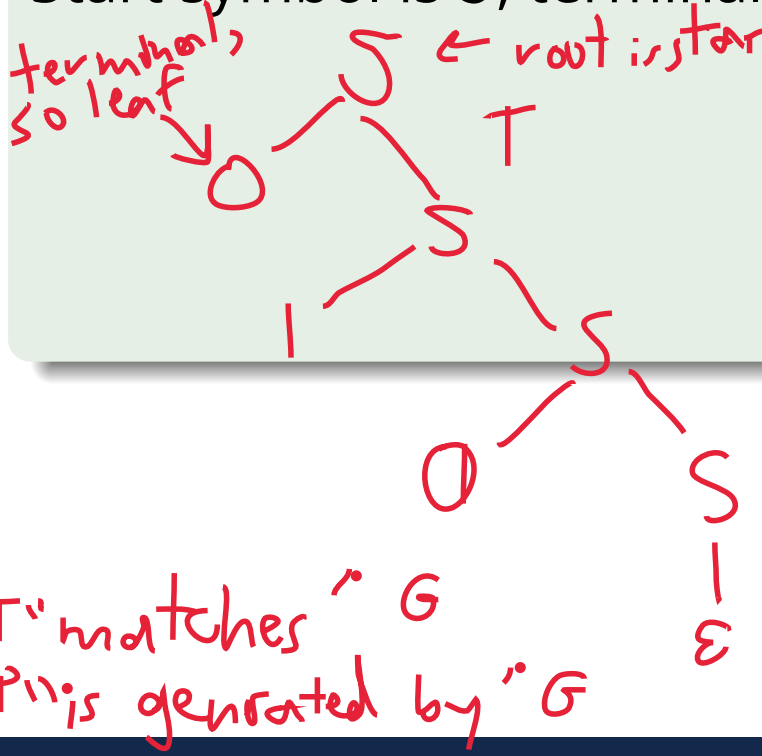
Example 1: Binary Strings

Start symbol is S, terminals are 0, 1, and ε, rules are:



G

$S \to 0S$

$S \to 1S$

$S \to \varepsilon$

010:
S
0S
01S
010S
010ε
010

T "matches" G
T is generated by G

string ≡ leaves read from left to right

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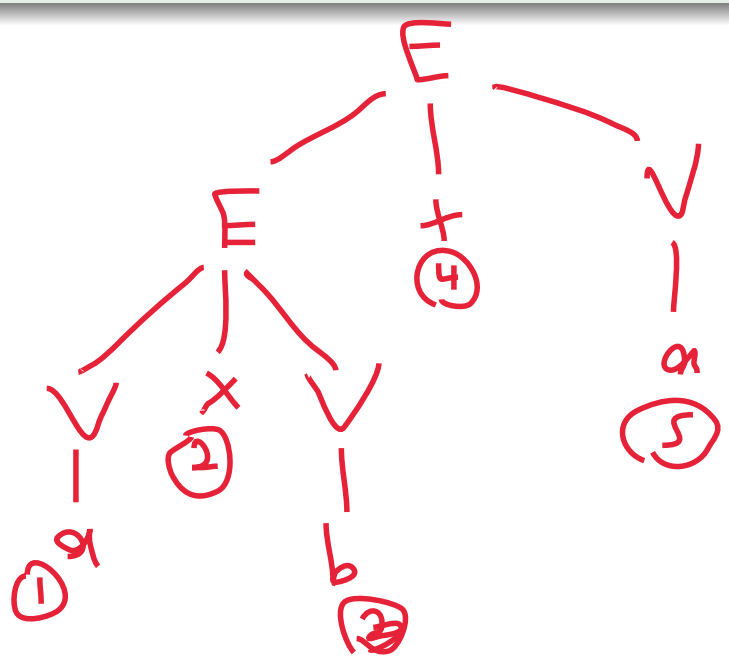
Example 2: Simple Arithmetic Expressions

Start symbols are E and V (also non-terminals), terminals are a, b, $+$, and $\times$, rules are:

$a \times b + a$

$$E \to E + V \mid E \times V \mid V + V \mid V \times V$$
$$V \to a \mid b$$

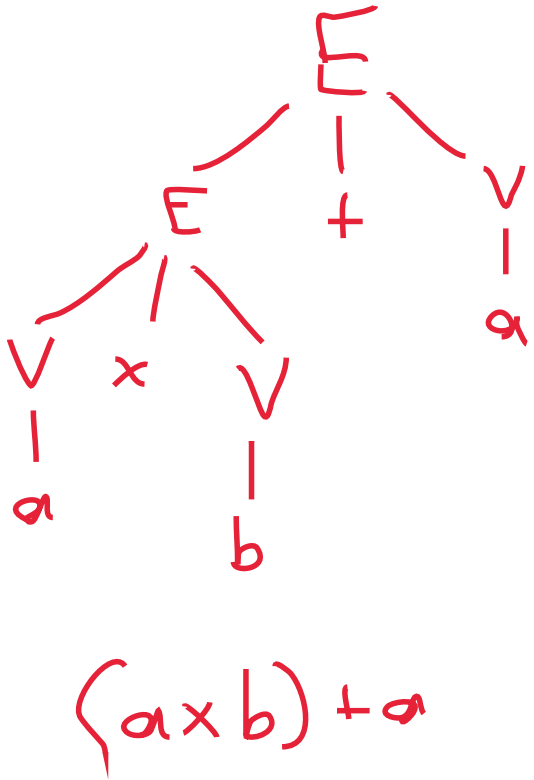


Ambiguity

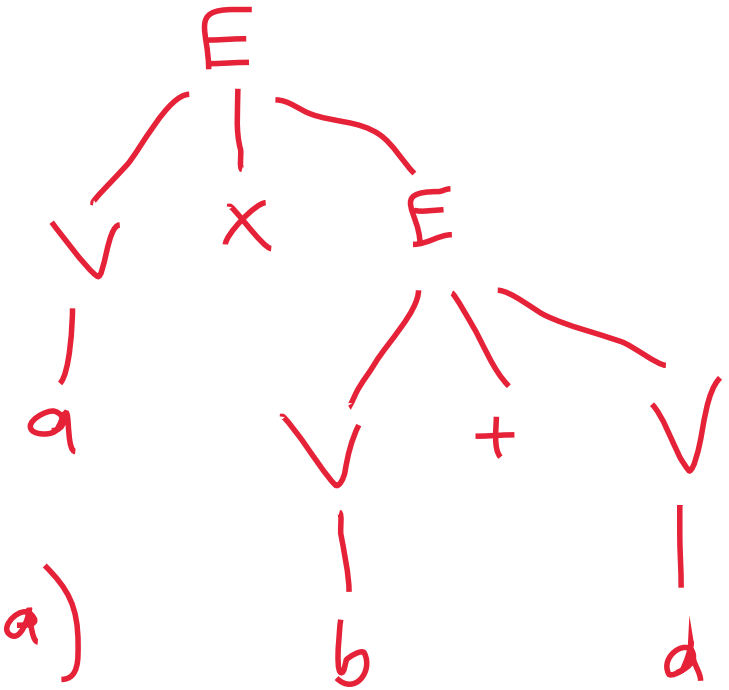
Example 3: Where are the implicit parentheses?

a x b+a

$$E \rightarrow E + V \mid E \times V \mid \mathbf{V \times E} \mid V + V \mid V \times V$$
$$V \rightarrow a \mid b$$



(a x b) + a



a x (b + a)

Designing Parse Trees, and Impossible Strings

$\{A, B, C, S\}$ $\{a, b, c\}$

$S \rightarrow AB \mid C$

$A \rightarrow Aa \mid a$

$B \rightarrow Bb \mid b$

$C \rightarrow C$ ~~C~~ c

Designing Parse Trees, and Impossible Strings

$$S \to AB \mid C$$
$$A \to Aa \mid a$$
$$B \to Bb \mid b$$
$$C \to \not{C}c$$

Can G generate the following strings? If so, give a parse tree. If not, explain why not.

- $aabca$ No. If a string contains a c, then it cannot contain a's or b's.

Designing Parse Trees, and Impossible Strings

$$S \to AB \mid C$$
$$A \to Aa \mid a \mid \varepsilon$$
$$B \to Bb \mid b$$
$$C \to c$$

$\{c\} \cup \{a^n b^m : n, m \in \mathbb{Z}, n \geq 0, m \geq 1\}$

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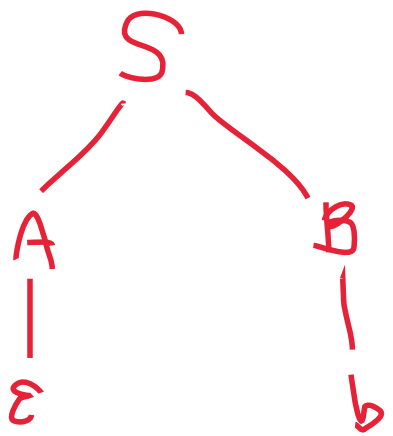
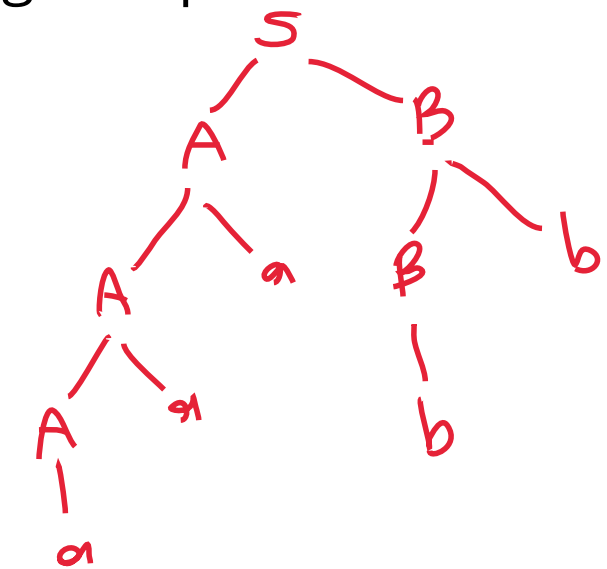
- $aabca$
- b Yes.
- aaabb





Recap: Learning Objectives

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