Introduction to Trees

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Learning Objectives

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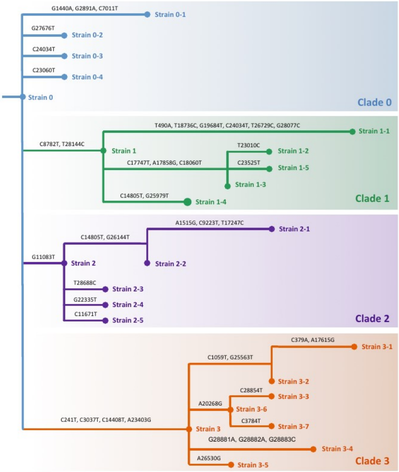
- Define and identify various tree properties.

Why do we care about trees?



A phylogenetic tree (Source)

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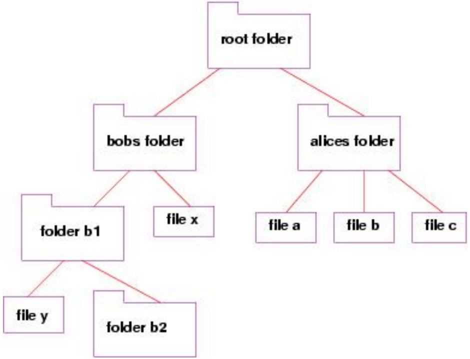

A file tree (Source)

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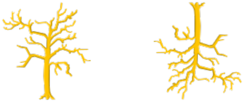
- Terms borrowed from biological trees and family trees



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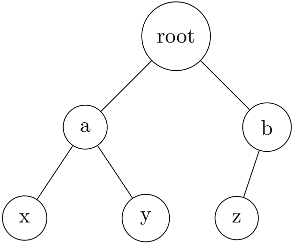

m-ary trees

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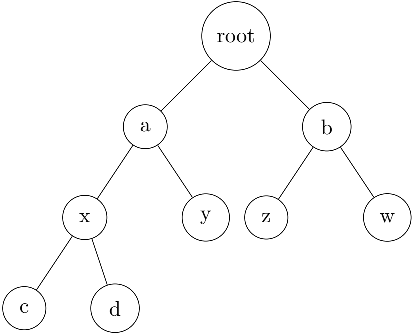

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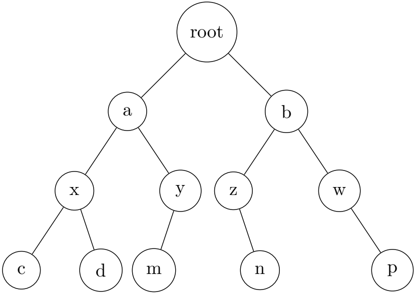

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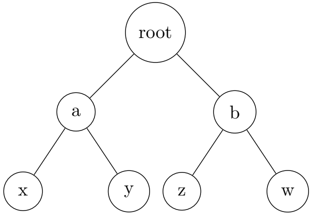
- full *and* complete m-ary tree



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Proof: Ask everyone how many kids they have (then add the root).

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Fact

The height of a full and complete binary tree with n nodes is proportional to $\log_2 n$.

Recap: Learning Objectives

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- Define and use tree terminology.
- Define and identify various tree properties.