Induction, Episode VI: Return of the I.H.

Part c: Proving Closed Forms by Induction

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Learning Objective

By the end of this lesson, you will be able to:

- Use induction to prove facts about a recursively defined function, e.g., that it has some specific closed form.

Inductive Proof on Recursive Definition

Example 1: Our Old Friend Visits Again

Define $g : \mathbb{Z}^+ \to \mathbb{Z}^+$ by

$$g(n) = \begin{cases} 2 & \text{if } n = 1 \\ n(n+1) + g(n-1) & \text{otherwise.} \end{cases}$$

Prove the closed-form expression for $g(n)$ is $\frac{n(n+1)(n+2)}{3}$.

Inductive Proof on Recursive Definition

Example 2: Odd Fibonacci

Recall the Fibonacci sequence defined by

$$F(0) = F(1) = 1$$
$$F(n) = F(n-1) + F(n-2)\ \forall n \geq 2.$$

Prove $F(3n+1)$ is odd for all $n \in \mathbb{N}$.

Recap: Learning Objective

By the end of this lesson, you will be able to:

- Use induction to prove facts about a recursively defined function, e.g., that it has some specific closed form.