Induction, Episode VI: Return of the I.H.

Part a: Recursive Definition

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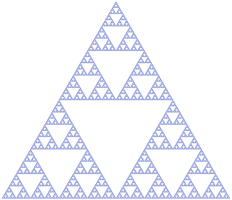
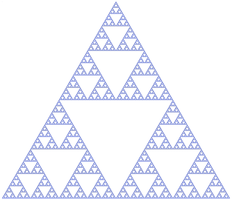


Sierpinski triangle

Droste effect

Recursive Definition

- Recursion: divide problem into smaller problem(s)
- Coding (mergesort and other "divide-and-conquer" algorithms)
- Art (esp. fractal art)
- Closely related to induction



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Example 2: Fibonacci

Define $F : \mathbb{N} \to \mathbb{N}$ by $F(0) = 0$, $F(1) = 1$, and
$F(n) = F(n-1) + F(n-2)$ for $n > 1$.

Preview: Connection to Induction

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Example 3: Implicit Summation

Define $g : \mathbb{Z}^+ \to \mathbb{Z}^+$ by

$$g(n) = \begin{cases} 2 & \text{if } n = 1 \\ n(n+1) + g(n-1) & \text{otherwise.} \end{cases}$$

Recap: Learning Objectives

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