Induction, Episode V: The Recursion Fairy Strikes Back

Part a: Meet The Recursion Fairy

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Learning Objectives

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- Explain the recursion fairy analogy of induction.

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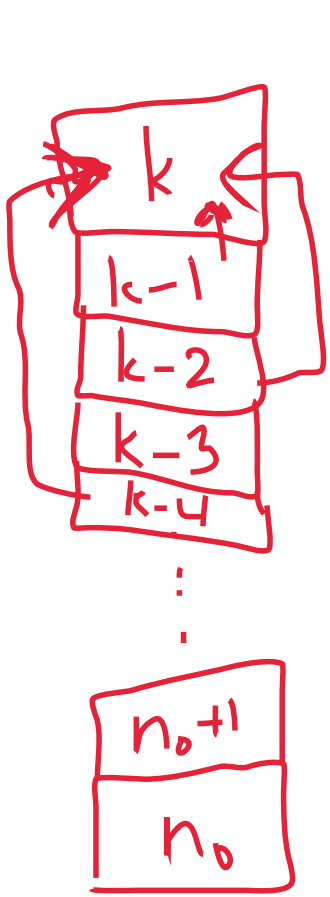

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$17 = 5 + 3 + 3 + 3 + 3$

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$$\left[\left[\bigwedge_{n=n_0}^{n_1} S(n)\right] \wedge \left[\forall k > n_1 \left[\left[\bigwedge_{n=n_0}^{k-1} S(n)\right] \rightarrow S(k)\right]\right]\right] \rightarrow \forall n \geq n_0\, S(n).$$

$n_1 = 12$, $n_0 = 8$, $k-1, k-2, k-4$, $n_0 = 0$, $k=1$: $k-4 = -3$

Base cases

Claim

for n_0 to n_1

Proof. The proof is by induction on n, the number of seats.

Base cases:

$n=8$: $8 = 3+5$ ✓

$n=9$: $9 = 3 \cdot 3$ ✓

$n=10$: $10 = 2 \cdot 5$ ✓

$n=11$: $11 = 5 + 2 \cdot 3$ ✓

$n=12$: $12 = 4 \cdot 3$ ✓

Let $k > 12$ be an arb. integer.

Suppose we can fill a tour bus with $n=8$ up to $n=k-1$ seats using 3s & 5s.

Put a group of size 5 on the bus.

Then there are $k-5$ seats left. Since $k>12$, $k-5 > 7$, or $k-5 \geq 8$. But also $k-5 < k$, so by I.H.

Recap: Learning Objectives

By the end of this lesson, you will be able to:

- Explain the recursion fairy analogy of induction.