# Induction, Episode IV: A New Proof Technique Part b: A Full, Slow Example

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- Use induction to prove a formula is correct for all integers starting at some  $n_0$ .

## Our First Inductive Proof Together

#### Claim

For every natural number n,  $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ .

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For every natural number n,  $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ .

#### Proof.

We prove the claim by induction on n.

Let P(n) be the statement  $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ .

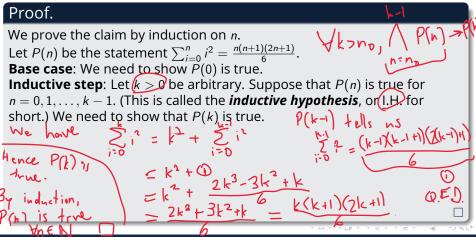
**Base case**: We need to show P(0) is true.

When 
$$n=0$$
, LHS is  $\frac{2}{100} = 0^2 = 0$ , and RHS is  $\frac{O(1)(1)}{6} = 0$ , so  $P(0)$  is true

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#### **Full Proof**

#### Proof.

We prove the claim by induction on n.  $P(n) = \sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ .

**Base case**: When n = 0,  $\sum_{i=0}^{0} i^2 = 0^2 = \frac{0(0+1)(2\cdot 0+1)}{6}$ , so P(0) is true. **Inductive step**: Let k > 0 be arbitrary. Suppose that P(n) is true for n = 0 through n = k - 1. We have

$$\sum_{i=0}^{k} i^2 = k^2 + \sum_{i=0}^{k-1} i^2$$

$$= k^2 + \frac{(k-1)(k-1+1)(2(k-1)+1)}{6}$$

$$= \frac{2k^3 + 3k^2 + k}{6}$$

$$= \frac{k(k+1)(2k+1)}{6}$$
(6)

(by the I.H.)

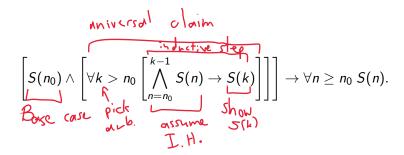
(by algebra

(by factoring).

Hence P(k) is true.

By induction, P(n) is true for all  $n \in \mathbb{N}$ .

### Reminder: Parts of Inductive Proof



By the end of this lesson, you will be able to:

- Given a claim, identify/state the key parts of an inductive proof.
- Use induction to prove a formula is correct for all integers starting at some  $n_0$ .