Two-way Bounding

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Learning Objective

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- Understand the difference between an exact result, an upper bound, and a lower bound.

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- Example: How many prime numbers are there between 100 and 1000?
- Could check all of them and find 143 primes (***exact result***)
- Easy ***lower bound***: 1, since we know 173 is prime
- Easy ***upper bound***: $(1000 - 100)/2 = 450$ (none of the even integers in that range can be prime)

$1 \leq$ # 3-digit primes ≤ 450

lower bound

upper bound

tight

Example: COVID-19 Restaurant Seating

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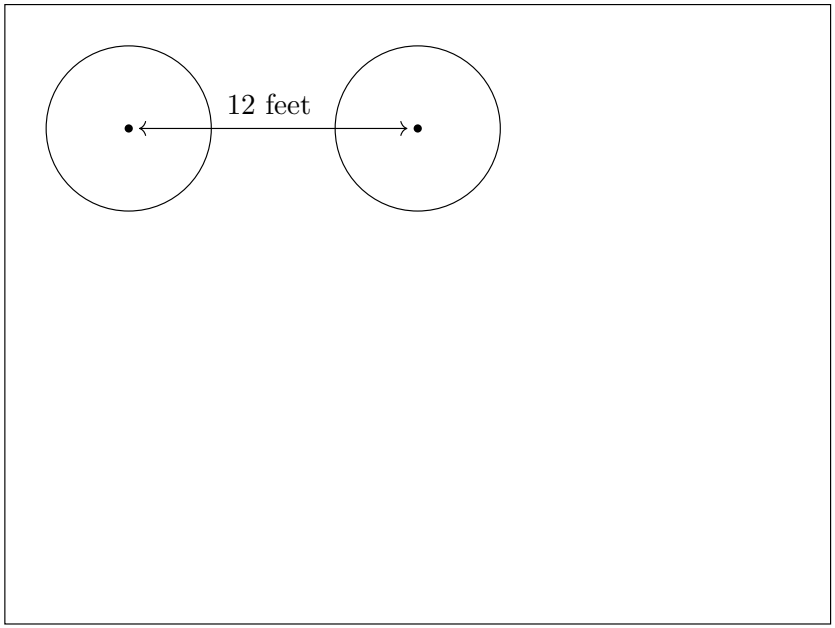
- Given: rectangle representing restaurant seating area



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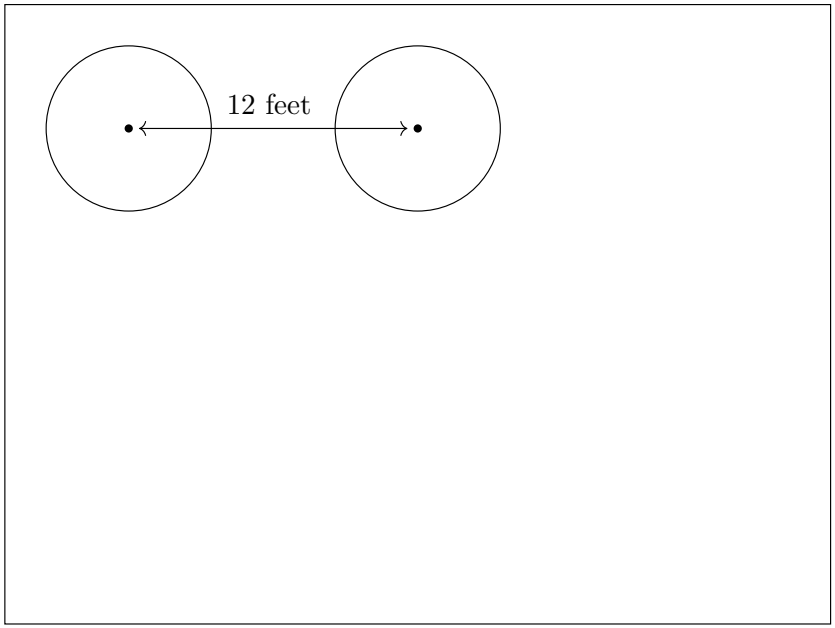
- Given: rectangle representing restaurant seating area
- Find: t, the maximum number of tables possible while complying with CDC physical distancing guidelines



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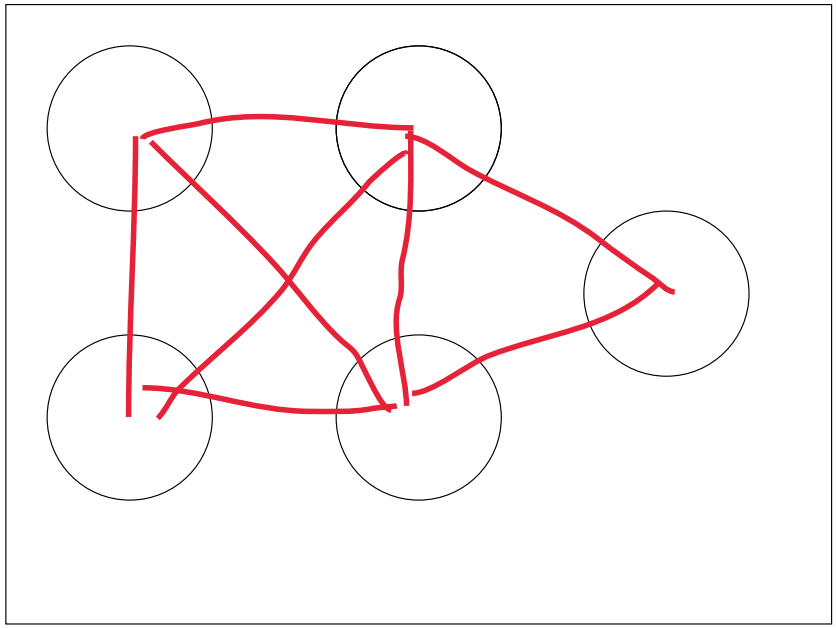

$t \geq 5$

$5 \leq t \leq 12$

LB: $5 \leq t$

UB: $\frac{\text{area of rectangle}}{\text{area of table}} \approx 12.3 \Rightarrow t \leq 12$

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- Given: rectangle representing restaurant seating area
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$5 \leq t \leq 8$

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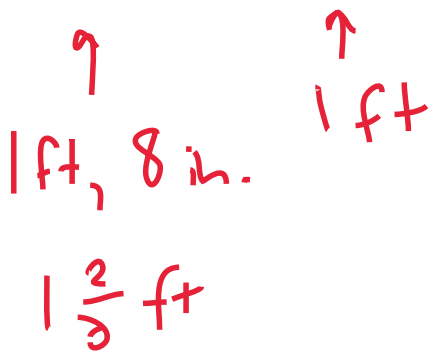

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- Given: SUV trunk with dimensions 4.5 ft $\times$ 4 ft $\times$ 3 ft, and 12 identical 20 in. $\times$ 12 in. $\times$ 3 ft moving boxes
- Find: b, the maximum number of boxes you can move in one trip

Example: Moving Boxes

$\frac{5}{3} \times 1 \times 3 = 5 \text{ ft}^3$

54 ft^3

- Given: SUV trunk with dimensions 4.5 ft $\times$ 4 ft $\times$ 3 ft, and 12 identical 20 in. $\times$ 12 in. $\times$ 3 ft moving boxes
- Find: b, the maximum number of boxes you can move in one trip
- Bound #1: volumes

$b \leq \left\lfloor \frac{54}{5} \right\rfloor = 10$

Example: Moving Boxes

- Given: SUV trunk with dimensions 4.5 ft $\times$ 4 ft $\times$ 3 ft, and 12 identical 20 in. $\times$ 12 in. $\times$ 3 ft moving boxes
- Find: b, the maximum number of boxes you can move in one trip
- Bound #1: volumes $b \leq 10$
- Bound #2: first attempt at packing $10 \leq b$

$10 \leq b \leq 10$

Conclude: $b = 10$

Recap: Learning Objective

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