Graph Isomorphisms

Ian Ludden

Learning Objectives

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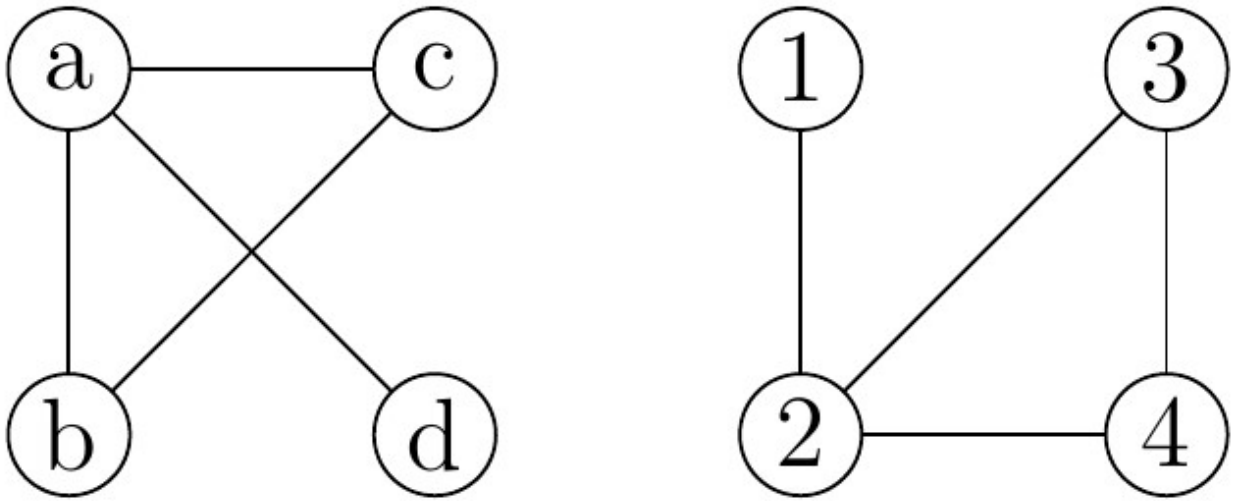
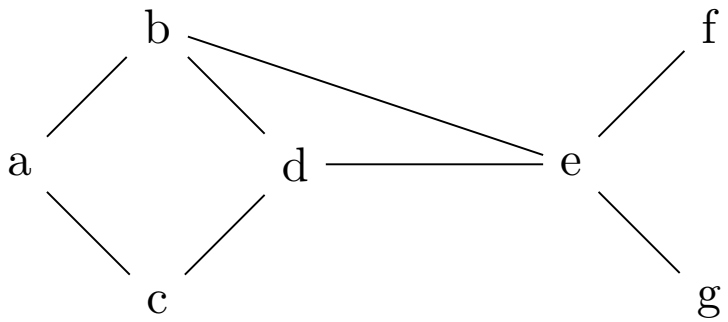
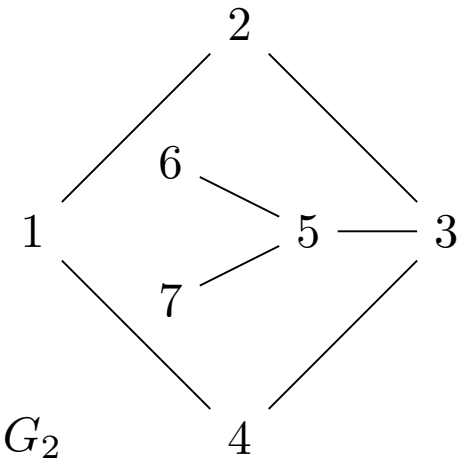


Counting isomorphisms



G_1



G_2

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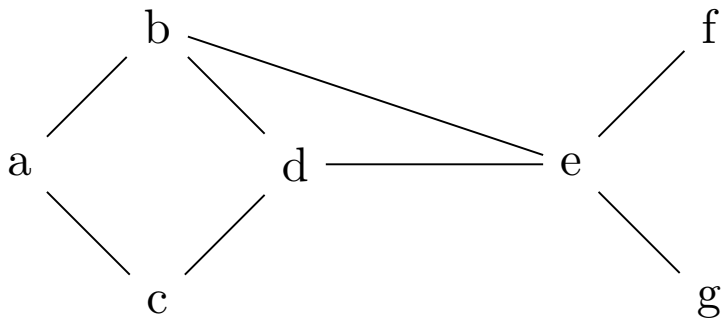

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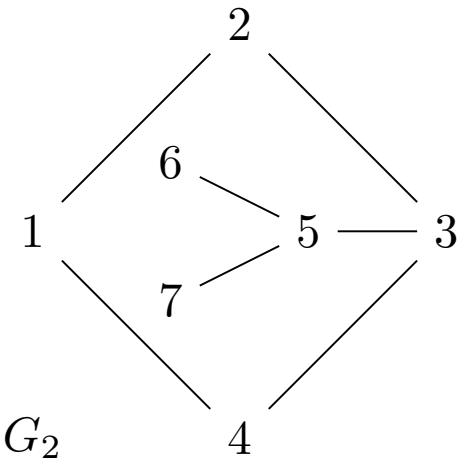

G_2

v	$f(v)$
a	
b	
c	
d	
e	
f	
g	

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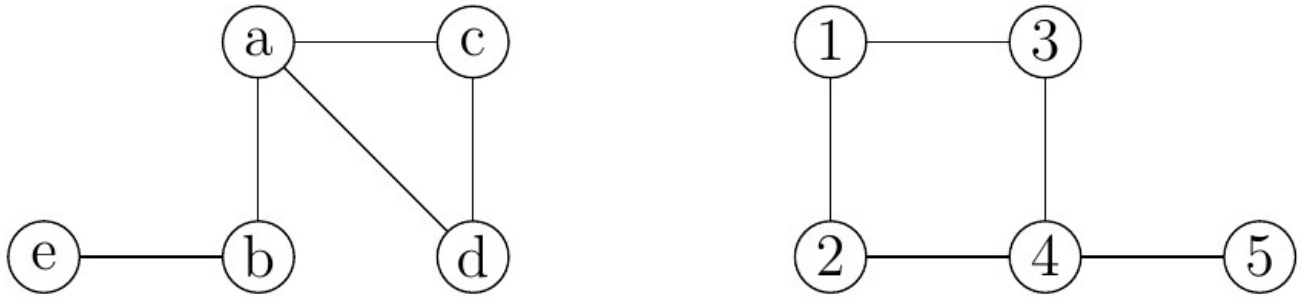

Recap: Learning Objectives

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