Special Graphs and Intermediate Definitions

Ian Ludden

Learning Objectives

By the end of this lesson, you will be able to:

Learning Objectives

By the end of this lesson, you will be able to:

- Define and identify K_n, C_n, W_n, and $K_{n,m}$.

Learning Objectives

By the end of this lesson, you will be able to:

- Define and identify K_n, C_n, W_n, and $K_{n,m}$.
- Recall definitions related to “moving around” on graphs.

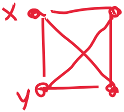
Special Types (Classes) of Graphs

What if we have...

$n = |V|$

- n vertices and every possible edge?

$K_n : |V| = n, \ |E| = \binom{n}{2} = n(n-1)$



K_4

Special Types (Classes) of Graphs

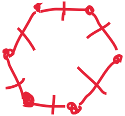
K_n

What if we have...

- n vertices and every possible edge?
- a cycle on n vertices? C_n

$|V| = n$, $|E| = n$

C_6



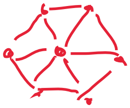
C_3

Special Types (Classes) of Graphs

What if we have...

- *n* vertices and every possible edge?
- a cycle on *n* vertices?
- a cycle on *n* vertices, but there's also a hub vertex?

W_n : $|V| = n+1$, $|E| = n+n = 2n$

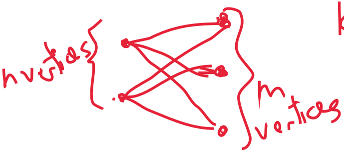


W_6

Special Types (Classes) of Graphs

What if we have...

- n vertices and every possible edge?
- a cycle on n vertices?
- a cycle on n vertices, but there's also a hub vertex?
- two separate sets of vertices and every possible edge between?



$K_{2,3}$

$K_{n,m}$: $|V| = n+m$

$|E| = n \cdot m$

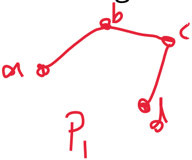
Walks, Paths, and Cycles

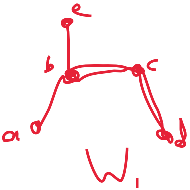
- Walks can repeat vertices/edges and are open or closed

Walks, Paths, and Cycles

- Walks can repeat vertices/edges and are open or closed
- Paths can't repeat vertices/edges

$\{a, b\}$





P_1 : ab-bc-cd

W_1 : ab-bc-cd-dc-cb-be

Walks, Paths, and Cycles

- Walks can repeat vertices/edges and are open or closed
- Paths can't repeat vertices/edges
- Cycles: C_n shows up as a ***subgraph***

W_6

$H = (V', E')$ is a subgraph of $G = (V, E)$ if

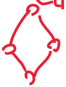
$V' \subseteq V$

$E' \subseteq E$

and E' only uses vertices in V'.

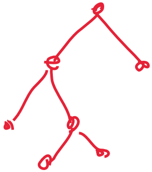
C_3

C_4



Walks, Paths, and Cycles

- Walks can repeat vertices/edges and are open or closed
- Paths can't repeat vertices/edges
- Cycles: C_n shows up as a ***subgraph***
- G is ***acyclic*** if no cycles as subgraphs

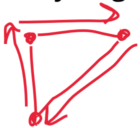


Walks, Paths, and Cycles

- Walks can repeat vertices/edges and are open or closed
- Paths can't repeat vertices/edges
- Cycles: C_n shows up as a ***subgraph***
- G is ***acyclic*** if no cycles as subgraphs
- ***Euler circuit***: closed walk that travels every edge exactly once

"Oil"-er

ends where it starts

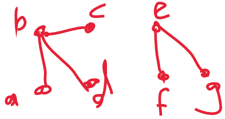


K_3 (same as C_3)

$\exists$ Euler ckt. in $G \longleftrightarrow G$ is connected and all degrees are even.

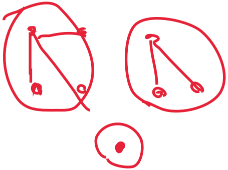
Stay connected

- G is ***connected*** if you can get anywhere from anywhere



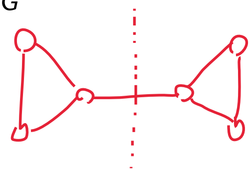
Stay connected

- *G* is ***connected*** if you can get anywhere from anywhere
- “islands” called ***connected components***



Stay connected

- *G* is ***connected*** if you can get anywhere from anywhere
- "islands" called ***connected components***
- ***cut edge***, if removed, would disconnect *G*

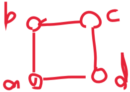


How far is it?

- The ***length*** of a walk/path is the number of edges

ab - bc - cd - da - ab - ba - ad

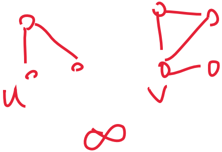
1 2 3 4 5 6 7



C_4

How far is it?

- The ***length*** of a walk/path is the number of edges
- The ***distance*** from u to v is the length of the shortest path



How far is it?

- The ***length*** of a walk/path is the number of edges
- The ***distance*** from u to v is the length of the shortest path
- The ***diameter*** is the max distance over all pairs of vertices

$$\max_{u,v \in V} \{ d(u,v) \}$$

$$d(u,u) = 0$$

Recap: Learning Objectives

By the end of this lesson, you will be able to:

- Define and identify K_n, C_n, W_n, and $K_{n,m}$. bipartite
- Recall definitions related to “moving around” on graphs.